ABSOLUTE MOTION ANALYSIS

Today’s Objective:
Students will be able to:
1. Determine the velocity and acceleration of a rigid body undergoing general plane motion using an absolute motion analysis.

In-Class Activities:
- Check Homework
- Reading Quiz
- Applications
- General Plane Motion
- Concept Quiz
- Group Problem Solving
- Attention Quiz
1. A body subjected to general plane motion undergoes a/an
   A) translation.
   B) rotation.
   C) simultaneous translation and rotation.
   D) out-of-plane movement.

2. In general plane motion, if the rigid body is represented by a slab, the slab rotates
   A) about an axis perpendicular to the plane.
   B) about an axis parallel to the plane.
   C) about an axis lying in the plane.
   D) None of the above.
The dumping bin on the truck rotates about a fixed axis passing through the pin at A. It is operated by the extension of the hydraulic cylinder BC.

The angular position of the bin can be specified using the angular position coordinate $\theta$ and the position of point C on the bin is specified using the coordinate $s$.

As a part of the design process for the truck, an engineer had to relate the velocity at which the hydraulic cylinder extends and the resulting angular velocity of the bin.
The large window is opened using a hydraulic cylinder AB.

The position B of the hydraulic cylinder rod is related to the angular position, \( \theta \), of the window.

A designer has to relate the translational velocity at B of the hydraulic cylinder and the angular velocity and acceleration of the window? How would you go about the task?
The position of the piston, $x$, can be defined as a function of the angular position of the crank, $\theta$. By differentiating $x$ with respect to time, the velocity of the piston can be related to the angular velocity, $\omega$, of the crank. This is necessary when designing an engine.

The stroke of the piston is defined as the total distance moved by the piston as the crank angle varies from 0 to 180°. How does the length of crank AB affect the stroke?
The absolute motion analysis method relates the position of a point, B, on a rigid body undergoing rectilinear motion to the angular position, $\theta$, of a line contained in the body.

Once a relationship in the form of $s_B = f(\theta)$ is established, the velocity and acceleration of point B are obtained in terms of the angular velocity and angular acceleration of the rigid body by taking the first and second time derivatives of the position function.

Usually the chain rule must be used when taking the derivatives of the position coordinate equation.
EXAMPLE I

Given: The platform is constrained to move vertically by the smooth vertical guides. The cam C rotates with a constant angular velocity, $\omega$.

Find: The velocity and acceleration of platform P as a function of the angle $\theta$ of cam C.

Plan: Use the fixed reference point O and define the position of the platform, P, in terms of the parameter $\theta$.

Take successive time derivatives of the position equation to find the velocity and acceleration.
Solution:

By geometry, \( y = r + r \sin \theta \)

By differentiating with respect to time,

\[
 v_P = r \cos \theta (\dot{\theta}) = r\omega \cos \theta
\]

Note that the cam rotates with a constant angular velocity.

Differentiating \( v_P \) to find the acceleration,

\[
 a_P = d(r\omega \cos \theta) / dt = r\omega (-\sin \theta) (\omega) = -r\omega^2 \sin \theta
\]
Given: Crank AB rotates at a constant velocity of $\omega = 150 \text{ rad/s}$.

Find: The velocity of point P when $\theta = 30^\circ$.

Plan: Define $x$ as a function of $\theta$ and differentiate with respect to time.
EXAMPLE II (continued)

Solution: \[ x_P = 0.2 \cos \theta + \sqrt{(0.75)^2 - (0.2 \sin \theta)^2} \]

\[ v_P = -0.2 \omega \sin \theta + (0.5)[(0.75)^2 - (0.2 \sin \theta)^2]^{0.5}(-2)(0.2 \sin \theta)(0.2 \cos \theta) \omega \]

\[ v_P = -0.2 \omega \sin \theta - [0.5(0.2)^2 \sin2\theta \omega] \sqrt{(0.75)^2 - (0.2 \sin \theta)^2} \]

At \( \theta = 30^\circ \), \( \omega = 150 \text{ rad/s} \) and \( v_P = -18.5 \text{ ft/s} = 18.5 \text{ ft/s} \)
CHECK YOUR UNDERSTANDING QUIZ

1. The position, s, is given as a function of angular position, \( \theta \), as \( s = 10 \sin 2\theta \). The velocity, v, is

   A) 20 cos 2\( \theta \)  
   B) 20 sin 2\( \theta \)  
   C) 20 \( \omega \) cos 2\( \theta \)  
   D) 20 \( \omega \) sin 2\( \theta \)

2. If \( s = 10 \sin 2\theta \), the acceleration, a, is

   A) 20 \( \alpha \) sin 2\( \theta \)  
   B) 20 \( \alpha \) cos 2\( \theta \) – 40 \( \omega \)\(^2\) sin 2\( \theta \)  
   C) 20 \( \alpha \) cos 2\( \theta \)  
   D) -40 \( \alpha \) sin 2\( \theta \)
GROUP PROBLEM SOLVING

Given: The circular cam is rotating about the fixed point O with $\omega=4$ rad/s, $\alpha = 2$ rad/s$^2$ with the dimensions shown.

Find: The velocity and acceleration of the plate when $\theta=30^\circ$.

Plan: Set the coordinate $x$ to be the distance between O and the plate. Relate $x$ to the angular position, $\theta$. Then take time derivatives of the position to find the velocity and acceleration.
Solution:

1) Relate $x$, the distance between $O$ and the plate, to $\theta$.
   
   \[ x = 120 \sin \theta + 150 \text{ (mm)} \]

2) Take time derivatives of the position to find the velocity and acceleration.

\[ v_C = \frac{dx}{dt} = 120 \cos \theta (\dot{\theta}) = 120 \omega \cos \theta \text{ (mm/s)} \]

\[ a_C = \frac{dv_C}{dt} = 120 \alpha \cos \theta + 120 \omega(-\sin \theta)(\omega) \text{ (mm/s}^2) \]

When $\theta = 30^\circ$, $\omega = 4 \text{ rad/s}$, and $\alpha = 2 \text{ rad/s}^2$. Substituting,

\[ v_C = 120 \times 4 \cos30^\circ = 416 \text{ mm/s} \]

\[ a_C = 120 \times 2 \cos30^\circ + 120(4^2) \times (-\sin30^\circ) = -752 \text{ mm/s}^2 \]
ATTENTION QUIZ

1. The sliders shown below are confined to move in the horizontal and vertical slots. If \( v_A = 10 \text{ m/s} \), determine the connecting bar’s angular velocity when \( \theta = 30^\circ \).
   - A) 10 rad/s
   - B) 10 rad/s
   - C) 8.7 rad/s
   - D) 8.7 rad/s

2. If \( v_A = 10 \text{ m/s} \) and \( a_A = 10 \text{ m/s} \), determine the angular acceleration, \( \alpha \), when \( \theta = 30^\circ \).
   - A) 0 rad/s\(^2\)
   - B) -50.2 rad/s\(^2\)
   - C) -112 rad/s\(^2\)
   - D) -173 rad/s\(^2\)
End of the Lecture

Let Learning Continue