EQUATIONS OF MOTION: RECTANGULAR COORDINATES

Today’s Objectives:
Students will be able to:
1. Apply Newton’s second law to determine forces and accelerations for particles in rectilinear motion.

In-Class Activities:
• Check Homework
• Reading Quiz
• Applications
• Equations of Motion Using Rectangular (Cartesian) Coordinates
• Concept Quiz
• Group Problem Solving
• Attention Quiz
1. In dynamics, the friction force acting on a moving object is always ________

   A) in the direction of its motion.  B) a kinetic friction.
   C) a static friction.  D) zero.

2. If a particle is connected to a spring, the elastic spring force is expressed by $F = ks$. The “s” in this equation is the

   A) spring constant.
   B) un-deformed length of the spring.
   C) difference between deformed length and un-deformed length.
   D) deformed length of the spring.
If a man is trying to move a 100 lb crate, how large a force $F$ must he exert to start moving the crate? What factors influence how large this force must be to start moving the crate?

If the crate starts moving, is there acceleration present?

What would you have to know before you could find these answers?
Objects that move in air (or other fluid) have a drag force acting on them. This drag force is a function of velocity.

If the dragster is traveling with a known velocity and the magnitude of the opposing drag force at any instant is given as a function of velocity, can we determine the time and distance required for dragster to come to a stop if its engine is shut off? How?
RECTANGULAR COORDINATES
(Section 13.4)

The equation of motion, \( F = m \mathbf{a} \), is best used when the problem requires finding forces (especially forces perpendicular to the path), accelerations, velocities, or mass. Remember, unbalanced forces cause acceleration!

Three scalar equations can be written from this vector equation. The equation of motion, being a vector equation, may be expressed in terms of its three components in the Cartesian (rectangular) coordinate system as

\[
\sum F = m \mathbf{a} \quad \text{or} \quad \sum F_x \mathbf{i} + \sum F_y \mathbf{j} + \sum F_z \mathbf{k} = m(a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k})
\]

or, as scalar equations, \( \sum F_x = ma_x \), \( \sum F_y = ma_y \), and \( \sum F_z = ma_z \).
PROCEDURE FOR ANALYSIS

• **Free Body Diagram (always critical!!)**

Establish your coordinate system and draw the particle’s free body diagram showing **only external forces**. These external forces usually include the weight, normal forces, friction forces, and applied forces. Show the ‘ma’ vector (sometimes called the inertial force) on a separate diagram.

Make sure any friction forces act opposite to the direction of motion! If the particle is connected to an elastic linear spring, a spring force equal to ‘ks’ should be included on the FBD.
PROCEDURE FOR ANALYSIS
(continued)

• Equations of Motion

If the forces can be resolved directly from the free-body diagram (often the case in 2-D problems), use the **scalar form** of the equation of motion. In more complex cases (usually 3-D), a Cartesian vector is written for every force and a **vector analysis** is often best.

A Cartesian vector formulation of the second law is

\[ \sum F = ma \quad \text{or} \]

\[ \sum F_x i + \sum F_y j + \sum F_z k = m(a_x i + a_y j + a_z k) \]

Three scalar equations can be written from this vector equation. You may only need two equations if the motion is in 2-D.
PROCEDURE FOR ANALYSIS
(continued)

• Kinematics

The second law only provides solutions for forces and accelerations. If velocity or position have to be found, kinematics equations are used once the acceleration is found from the equation of motion.

Any of the kinematics tools learned in Chapter 12 may be needed to solve a problem.

Make sure you use consistent positive coordinate directions as used in the equation of motion part of the problem!
EXAMPLE

Given: The 200 lb mine car is hoisted up the incline. The motor M pulls in the cable with an acceleration of 4 ft/s².

Find: The acceleration of the mine car and the tension in the cable.

Plan:

Draw the free-body and kinetic diagrams of the car.

Using a dependent motion equation, determine an acceleration relationship between cable and mine car.

Apply the equation of motion to determine the cable tension.
EXAMPLE
(continued)

Solution:

1) Draw the free-body and kinetic diagrams of the mine car:

Since the motion is up the incline, rotate the x-y axes. Motion occurs only in the x-direction. We are also neglecting any friction in the wheel bearings, etc., on the cart.
EXAMPLE (continued)

2) The cable equation results in

\[ s_p + 2 \ s_c = l_t \]

Taking the derivative twice yields

\[ a_p + 2 \ a_c = 0 \quad (\text{eqn. 1}) \]

The relative acceleration equation is

\[ a_p = a_c + \frac{a_p}{c} \]

As the motor is mounted on the car,

\[ a_{p/c} = 4 \ \text{ft/s}^2 \]

So,

\[ a_p = a_c + 4 \ \text{ft/s}^2 \quad (\text{eqn. 2}) \]

Solving equations 1 and 2, yields

\[ a_c = 1.333 \ \text{ft/s}^2 \]
3) Apply the equation of motion in the x-direction:

\[ \sum F_x = ma_x \Rightarrow 3T - mg(\sin30^\circ) = ma_x \]

\[ \Rightarrow 3T - (200)(\sin 30^\circ) = (200/32.2) (1.333) \]

\[ \Rightarrow T = 36.1 \text{ lb} \]
CHECK YOUR UNDERSTANDING QUIZ

1. If the cable has a tension of 3 N, determine the acceleration of block B.
   A) 4.26 m/s²  B) 4.26 m/s²  C) 8.31 m/s²  D) 8.31 m/s²

2. Determine the acceleration of the block.
   A) 2.20 m/s²  B) 3.17 m/s²  C) 11.0 m/s²  D) 4.26 m/s²
GROUP PROBLEM SOLVING

Given:  
\[ W_A = 10 \text{ lb} \]
\[ W_B = 20 \text{ lb} \]
\[ v_{oA} = 2 \text{ ft/s} \]
\[ \mu_k = 0.2 \]

Find: \[ v_A \] when A has moved 4 feet to the right.

Plan: This is not an easy problem, so think carefully about how to approach it!
GROUP PROBLEM SOLVING

Given:  

\[ W_A = 10 \ \text{lb} \]
\[ W_B = 20 \ \text{lb} \]
\[ v_{oA} = 2 \ \text{ft/s} \]
\[ \mu_k = 0.2 \]

Find:  \[ v_A \] when A has moved 4 feet to the right.

Plan: Since both forces and velocity are involved, this problem requires both the equation of motion and kinematics. First, draw free body diagrams of A and B. Apply the equation of motion to each.
Using dependent motion equations, derive a relationship between \( a_A \) and \( a_B \) and use with the equation of motion formulas.
GROUP PROBLEM SOLVING
(continued)

Solution:

Free-body and kinetic diagrams of B:

\[ W_B - 2T = m_B a_B \]

Apply the equation of motion to B:

\[ + \sum F_y = m a_y \]

\[ W_B - 2T = m_B a_B \]

\[ 20 - 2T = \frac{20}{32.2} a_B \]  
\[ (1) \]
GROUP PROBLEM SOLVING
(continued)

Free-body and kinetic diagrams of A:

Apply the equations of motion to A:

\[ \sum F_y = m a_y = 0 \]
\[ N = W_A = 10 \text{ lb} \]
\[ F = \mu_k N = 2 \text{ lb} \]

\[ \sum F_x = m a_x \]
\[ F - T = m_A a_A \]
\[ 2 - T = \frac{10}{32.2} a_A \quad (2) \]
Now consider the **kinematics**.

**Constraint equation:**

\[ s_A + 2 \ s_B = \text{constant} \]

or

\[ \nu_A + 2 \ \nu_B = 0 \]

Therefore

\[ a_A + 2 \ a_B = 0 \]

\[ a_A = -2 \ a_B \quad \text{(3)} \]

(Notice \( a_A \) is considered positive to the left and \( a_B \) is positive downward.)
Now combine equations (1), (2), and (3).

\[ T = \frac{22}{3} = 7.33 \text{ lb} \]

\[ a_A = -17.16 \text{ ft/s}^2 = 17.16 \text{ ft/s}^2 \rightarrow \]

Now use the kinematic equation:

\[ (v_A)^2 = (v_{0A})^2 + 2 \cdot a_A (s_A - s_{0A}) \]

\[ (v_A)^2 = (2)^2 + 2 \cdot (17.16)(4) \]

\[ v_A = 11.9 \text{ ft/s} \rightarrow \]
ATTENTION QUIZ

1. Determine the tension in the cable when the 400 kg box is moving upward with a 4 m/s² acceleration.
   A) 2265 N  B) 3365 N  C) 5524 N  D) 6543 N

2. A 10 lb particle has forces of \( F_1 = (3i + 5j) \) lb and \( F_2 = (-7i + 9j) \) lb acting on it. Determine the acceleration of the particle.
   A) \((-0.4i + 1.4j\) ft/s²  B) \((-4i + 14j)\) ft/s²
   C) \((-12.9i + 45j)\) ft/s²  D) \((13i + 4j)\) ft/s²
End of the Lecture

Let Learning Continue