ABSOLUTE DEPENDENT MOTION ANALYSIS OF TWO PARTICLES

Today’s Objectives:
Students will be able to:
1. Relate the positions, velocities, and accelerations of particles undergoing dependent motion.

In-Class Activities:
• Check Homework
• Reading Quiz
• Applications
• Define Dependent Motion
• Develop Position, Velocity, and Acceleration Relationships
• Concept Quiz
• Group Problem Solving
• Attention Quiz
READING QUIZ

1. When particles are interconnected by a cable, the motions of the particles are ______
   A) always independent.               B) always dependent.
   C) not always dependent.              D) None of the above.

2. If the motion of one particle is dependent on that of another particle, each coordinate axis system for the particles ______
   A) should be directed along the path of motion.
   B) can be directed anywhere.
   C) should have the same origin.
   D) None of the above.
APPLICATIONS

The cable and pulley system shown can be used to modify the speed of the mine car, A, relative to the speed of the motor, M.

It is important to establish the relationships between the various motions in order to determine the power requirements for the motor and the tension in the cable.

For instance, if the speed of the cable (P) is known because we know the motor characteristics, how can we determine the speed of the mine car? Will the slope of the track have any impact on the answer?
Rope and pulley arrangements are often used to assist in lifting heavy objects. The total lifting force required from the truck depends on both the weight and the acceleration of the cabinet.

How can we determine the acceleration and velocity of the cabinet if the acceleration of the truck is known?
DEPENDENT MOTION (Section 12.9)

In many kinematics problems, the motion of one object will depend on the motion of another object.

The blocks in this figure are connected by an inextensible cord wrapped around a pulley. If block A moves downward along the inclined plane, block B will move up the other incline.

The motion of each block can be related mathematically by defining position coordinates, $s_A$ and $s_B$. Each coordinate axis is defined from a fixed point or datum line, measured positive along each plane in the direction of motion of each block.
In this example, position coordinates $s_A$ and $s_B$ can be defined from fixed datum lines extending from the center of the pulley along each incline to blocks A and B.

If the **cord has a fixed length**, the position coordinates $s_A$ and $s_B$ are **related mathematically** by the equation

$$s_A + l_{CD} + s_B = l_T$$

Here $l_T$ is the total cord length and $l_{CD}$ is the length of cord passing over the arc CD on the pulley.
The velocities of blocks A and B can be related by differentiating the position equation. Note that $l_{CD}$ and $l_T$ remain constant, so $dl_{CD}/dt = dl_T/dt = 0$

\[
\frac{ds_A}{dt} + \frac{ds_B}{dt} = 0 \implies v_B = -v_A
\]

The negative sign indicates that as A moves down the incline (positive $s_A$ direction), B moves up the incline (negative $s_B$ direction).

**Accelerations** can be found by differentiating the velocity expression. Prove to yourself that $a_B = -a_A$. 

**DEPENDENT MOTION**
(continued)
DEPENDENT MOTION EXAMPLE

Consider a more complicated example. Position coordinates \((s_A\) and \(s_B\)) are defined from fixed datum lines, measured along the direction of motion of each block.

Note that \(s_B\) is only defined to the center of the pulley above block B, since this block moves with the pulley. Also, \(h\) is a constant.

The red colored segments of the cord remain constant in length during motion of the blocks.
DEPENDENT MOTION EXAMPLE (continued)

The position coordinates are related by the equation
\[ 2s_B + h + s_A = l_T \]

Where \( l_T \) is the total cord length minus the lengths of the red segments.

Since \( l_T \) and \( h \) remain constant during the motion, the velocities and accelerations can be related by two successive time derivatives:
\[ 2v_B = -v_A \quad \text{and} \quad 2a_B = -a_A \]

When block B moves downward \((+s_B)\), block A moves to the left \((-s_A)\). Remember to be consistent with your sign convention!
This example can also be worked by defining the position coordinate for B (s_B) from the bottom pulley instead of the top pulley.

The position, velocity, and acceleration relations then become:

\[
2(h - s_B) + h + s_A = l_T
\]

and

\[
2v_B = v_A \quad 2a_B = a_A
\]

Prove to yourself that the results are the same, even if the sign conventions are different than the previous formulation.
DEPENDENT MOTION: PROCEDURES

These procedures can be used to relate the dependent motion of particles moving along rectilinear paths (only the magnitudes of velocity and acceleration change, not their line of direction).

1. Define position coordinates from fixed datum lines, along the path of each particle. Different datum lines can be used for each particle.

2. Relate the position coordinates to the cord length. Segments of cord that do not change in length during the motion may be left out.

3. If a system contains more than one cord, relate the position of a point on one cord to a point on another cord. Separate equations are written for each cord.

4. Differentiate the position coordinate equation(s) to relate velocities and accelerations. Keep track of signs!
EXAMPLE

Given: In the figure on the left, the cord at A is pulled down with a speed of 2 m/s.

Find: The speed of block B.

Plan: There are two cords involved in the motion in this example. There will be two position equations (one for each cord). Write these two equations, combine them, and then differentiate them.
Solution:

1) Define the position coordinates from a fixed datum line. Three coordinates must be defined: one for point A \( (s_A) \), one for block B \( (s_B) \), and one for block C \( (s_C) \).

- Define the datum line through the top pulley (which has a fixed position).
- \( s_A \) can be defined to the point A.
- \( s_B \) can be defined to the center of the pulley above B.
- \( s_C \) is defined to the center of pulley C.
- All coordinates are defined as positive down and along the direction of motion of each point/object.
EXAMPLE (continued)

2) Write position/length equations for each cord. Define $l_1$ as the length of the first cord, minus any segments of constant length. Define $l_2$ in a similar manner for the second cord:

Cord 1: $s_A + 2s_C = l_1$
Cord 2: $s_B + (s_B - s_C) = l_2$

3) Eliminating $s_C$ between the two equations, we get

$$s_A + 4s_B = l_1 + 2l_2$$

4) Relate velocities by differentiating this expression. Note that $l_1$ and $l_2$ are constant lengths.

$$v_A + 4v_B = 0 \implies v_B = -0.25v_A = -0.25(2) = -0.5 \text{ m/s}$$

The velocity of block B is 0.5 m/s up (negative $s_B$ direction).
CONCEPT QUIZ

1. Determine the speed of block B.
   A) 1 m/s       B) 2 m/s
   C) 4 m/s       D) None of the above.

2. Two blocks are interconnected by a cable. Which of the following is correct?
   A) \( v_A = -v_B \)       B) \( (v_x)_A = -(v_x)_B \)
   C) \( (v_y)_A = -(v_y)_B \) D) All of the above.
GROUP PROBLEM SOLVING

Given: In this pulley system, block A is moving downward with a speed of 4 ft/s while block C is moving up at 2 ft/s.

Find: The speed of block B.

Plan:

All blocks are connected to a single cable, so only one position/length equation will be required. Define position coordinates for each block, write out the position relation, and then differentiate it to relate the velocities.
GROUP PROBLEM SOLVING

Solution:

1) A datum line can be drawn through the upper, fixed, pulleys and position coordinates defined from this line to each block (or the pulley above the block).

2) Defining $s_A$, $s_B$, and $s_C$ as shown, the position relation can be written:

$$s_A + 2s_B + s_C = 1$$

3) Differentiate to relate velocities:

$$v_A + 2v_B + v_C = 0$$

$$\Rightarrow 4 + 2v_B + (-2) = 0$$

$$\Rightarrow v_B = -1 \text{ ft/s}$$

The velocity of block B is 1 ft/s up (negative $s_B$ direction).
ATTENTION QUIZ

1. Determine the speed of block B when block A is moving down at 6 ft/s while block C is moving down at 18 ft/s.

   A) 24 ft/s    B) 3 ft/s
   C) 12 ft/s    D) 9 ft/s

2. Determine the velocity vector of block A when block B is moving downward with a speed of 10 m/s.

   A) (8\textbf{i} + 6\textbf{j}) m/s    B) (4\textbf{i} + 3\textbf{j}) m/s
   C) (-8\textbf{i} - 6\textbf{j}) m/s    D) (3\textbf{i} + 4\textbf{j}) m/s
End of the Lecture

Let Learning Continue