PART I. Do not use a calculator on this part.

Complete the following statements. (12 points)

1. The derivative of a function \( f \) is the function \( f' \) that is defined by \( f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \).

2. A function \( g \) is continuous at \( x = b \) if \( \lim_{x \to b} g(x) = g(b) \).

3. Let \( f, g, h \) be functions such that \( f(x) \leq g(x) \leq h(x) \) for all \( x \) in an interval \((-5,5)\). If \( \lim_{x \to 1} f(x) = \lim_{x \to 1} h(x) = 7 \), then \( \lim_{x \to 1} g(x) = 7 \) according to the Squeeze theorem.

4. If \( x < 1 \), then \( |x - 1| = -(x-1) \). (Write the answer in terms of \( x \) without using absolute value bars.)

5. Is the following statement true or false? If a function is continuous at \( x = 5 \), then it is also differentiable at \( x = 5 \).
   \[ \text{FALSE} \]

6. Determine \( \lim_{x \to 1} \frac{x}{1-x^2} \).

   \[ \text{As } x \to 1^-, \quad 1-x^2 \to 0^-; \quad \text{When } x < 1, \quad \frac{1}{1-x^2} > 0. \]

   \[ \text{Hence } \frac{x}{1-x^2} > 0. \quad \text{We conclude } \lim_{x \to 1} \frac{x}{1-x^2} = +\infty \]

7. Determine \( \lim_{x \to \infty} \frac{3-6x^2}{8x^2 + x - 5} \).

   \[ \lim_{x \to \infty} \frac{3-6x^2}{8x^2 + x - 5} = \lim_{x \to \infty} \frac{3}{8} \left(1 - \frac{1}{1 - \frac{5}{x^2}}\right) = \frac{3}{8} \]

8. Determine all vertical and horizontal asymptotes of \( f(x) = \frac{5x^2 + 1}{x^2 - 2x - 3} \).

   \[ f(x) = \frac{5x^2 + 1}{(x+1)(x-3)} \quad \text{Vertical asymptotes: } x = -1 \text{ and } x = 3 \]

   \[ \lim_{x \to \pm \infty} \frac{5x^2 + 1}{x^2 - 2x - 3} = \lim_{x \to \pm \infty} \frac{5 + \frac{1}{x^2}}{1 - \frac{2}{x} - \frac{3}{x^2}} = 5 \]

   \[ y = 5 \text{ is the horizontal asymptote.} \]
9. Let \( f(x) = \begin{cases} 6 - \cos x & \text{if } x < 0 \\ x^2 + 3b & \text{if } x > 0 \end{cases} \), where \( b \) denotes a constant. What would make \( f \) continuous at \( x = 0 \)? (6 pts)

\[
\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (6 - \cos x) = 6 - \cos 0 = 6 - 1 = 5.
\]
\[
\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (x^2 + 3b) = 3b.
\]
Thus \( \lim_{x \to 0} f(x) = 5 \) if \( 3b = 5 \).
Thus, the function \( f \) is continuous at \( x = 0 \) if
\[
6 = \frac{5}{3}
\]
and if the discontinuity at \( x = 0 \) is removed
by defining \( f(0) = 5 \).

10. Let \( f(x) = \cos^{-1} x \) and \( g(x) = x - 5 \). (6 pts)

(a) Find \((f \circ g)(x)\).

\[
(f \circ g)(x) = f(g(x)) = f(x-5) = \cos^{-1}(x-5).
\]

(b) Determine the interval on which \((f \circ g)(x)\) is continuous.

The function \( g(x) \) is continuous on \((-\infty, \infty)\). \( f(x) = \cos^{-1} x \) is continuous on \((-1, 1)\).
Thus, \( f \circ g \) is continuous at \( x \) if \( -1 < x-5 < 1 \).

Here, \( f \circ g \) is continuous on \((4, 6)\).

11. Let \( f(x) = \frac{5}{x+2} \). Use the definition of the derivative (NOT the rules) to compute the derivative function \( f' \). (8 pts)

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{5}{x+h+2} - \frac{5}{x+2}}{h}
\]

\[
= \lim_{h \to 0} \frac{1}{h} \left[ \frac{5}{x+h+2} - \frac{5}{x+2} \right] = \lim_{h \to 0} \frac{1}{h} \left[ \frac{5(x+2) - 5(x+h+2)}{(x+h+2)(x+2)} \right]
\]

\[
= \lim_{h \to 0} \frac{1}{h} \left[ \frac{5x+10 - 5x - 5h - 10}{(x+h+2)(x+2)} \right] = \lim_{h \to 0} \frac{1}{h} \left[ \frac{-5h}{(x+h+2)(x+2)} \right]
\]

\[
= \lim_{h \to 0} \frac{-5}{(x+h+2)(x+2)} = \frac{-5}{(x+2)^2}
\]

\[\therefore f'(x) = -\frac{5}{(x+2)^2}\]
12. Find the equation of the tangent line to \( f(x) = x^2 - 3x \) at \( x = -4 \).

When \( x = -4 \), \( y = f(-4) = (-4)^2 - 3(-4) = 16 + 12 = 28 \).

Differentiating, \( f'(x) = 2x - 3 \). The slope of the tangent line at \( x = -4 \) is \( f'(-4) = 2(-4) - 3 = -8 - 3 = -11 \).

The equation of the tangent line is \( y - 28 = -11(x + 4) \) or \( y = -11x + 16 \).

13. Use the rules to differentiate the following functions:

(a) \( y = 4x^9 - x \)

\[ y' = 36x^8 - 1 \]

(b) \( y = \sqrt{x} + \frac{3}{x^4} \)

\[ y' = \frac{dx}{dx} \left( x^{\frac{1}{2}} + 3x^{-4} \right) = \frac{-1}{2} x^{-\frac{3}{2}} - 12x^{-5} \]

\[ y' = \frac{1}{2\sqrt{x}} - \frac{12}{x^5} \]

(c) \( y = \frac{2x - 3}{x + 4} \)

By the Quotient Rule,

\[ y' = \frac{(x + 4)\frac{d}{dx}(2x - 3) - (2x - 3)\frac{d}{dx}(x + 4)}{(x + 4)^2} = \frac{2(x + 4) - (2x - 3)}{(x + 4)^2} = \frac{11}{(x + 4)^2} \]

14. Use the rules to find the velocity and acceleration functions of an object whose position function is \( s(t) = -16t^2 + 500 \).

Velocity:

\[ v = \frac{ds}{dt} = \frac{d}{dt} (-16t^2 + 500) = -32t \quad \Rightarrow \quad \text{\( v = -32t \)} \]

 Acceleration:

\[ a = \frac{dv}{dt} = \frac{d}{dt} (-32t) = -32 \quad \Rightarrow \quad \text{\( a = -32 \)} \]
PART II. (14 points) Use your calculator to work the following problems.

1. The function \( s(t) = t^3 + 4t \) represents the position in meters of an object at time \( t \) seconds. Compute the average velocity of the object between (a) \( t = 1.99 \) and \( t = 2 \), (b) \( t = 1.999 \) and \( t = 2 \), (c) \( t = 2.01 \) and \( t = 2 \), (d) \( t = 2.001 \) and \( t = 2 \). Estimate the instantaneous velocity at \( t = 2 \) seconds. What are the units of velocity?

\[
\text{Average velocity between times 2 and } t = \frac{s(t) - s(2)}{t - 2}.
\]

As was shown several times in class, \( s(t) \) can be done quickly on TI-89 by defining the functions:

- Define \( d(t) = t^3 + 4t \)
- Define \( d(t) = (s(t) - s(2)) / (t - 2) \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( d(t) )</th>
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<tbody>
<tr>
<td>1.99</td>
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<tr>
<td>1.999</td>
<td>15.994</td>
</tr>
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<tr>
<td>2.01</td>
<td>16.061</td>
</tr>
</tbody>
</table>

From the table,

a) \( 15.94 \text{ m/sec} \)

b) \( 15.99 \text{ m/sec} \)

c) \( 16.00 \text{ m/sec} \)

d) \( 16.01 \text{ m/sec} \)

Estimate of the instantaneous velocity at \( t = 2 \): (16 \( \text{ m/sec} \))

Units of velocity: meters per second.
2. Let \( f(x) = \sqrt{3 + \sin x} \). Numerically estimate the derivative \( f'(6) \) to three decimal places. Do this by using a difference quotient to numerically estimate the left-hand and right-hand derivatives at \( x = 6 \). Include a table showing at least 3 estimates of the left-hand derivative and at least 3 estimates of the right-hand derivative.

First define the function \( f \) on the TI-89:

Define \( f(x) = \sqrt{3 + \sin(x)} \).

Then define the difference quotient:

\[
\frac{d(x)}{x-6} = \frac{f(x) - f(6)}{(x-6)}.
\]

**Table:**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( d(x) )</th>
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<tbody>
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<tr>
<td>5.9999</td>
<td>0.29106</td>
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**Left-hand derivative**

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<th>( d(x) )</th>
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<tr>
<td>6.1</td>
<td>0.29222</td>
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<td>6.01</td>
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<tr>
<td>6.0001</td>
<td>0.29106</td>
</tr>
</tbody>
</table>

**Right-hand derivative**

From the tables, it appears that \( f'(6) = 0.291 \) to three decimal places.