**Problem** (Page 71 #33). Let $G$ be a group. Show that

$$Z(G) = \bigcap_{a \in G} C(a).$$

[This means the intersection of all subgroups of the form $C(a)$].

**Proof.**

[We show $Z(G) = \bigcap_{a \in G} C(a)$ by showing mutual set inclusion.]

Let $x \in Z(G)$. Then $x$ commutes with all $a \in G \implies x \in C(a) \ \forall \ a \in G$. Thus $Z(G) \subseteq \bigcap_{a \in G} C(a)$.

Now suppose $x \in \bigcap_{a \in G} C(a)$. Then $x$ commutes with every $a \in G$, so $x \in C(a) \ \forall \ a \in G$. Thus $\bigcap_{a \in G} C(a) \subseteq Z(G)$, and so $Z(G) = \bigcap_{a \in G} C(a)$ by mutual set inclusion. \qed