

Linear Models and Regression with TI-89

To illustrate this topic, we use Problem 18 on page 115 as an example. Enrollment in public colleges (in thousands) in selected years is shown in the table below.

| | | | | | | |
|-------------------|------|------|--------|--------|--------|-------|
| Year | 1980 | 1985 | 1990 | 1995 | 2000 | 2003 |
| Enrollment | 9457 | 9479 | 10,845 | 11,092 | 11,750 | 11894 |

(a) Find a linear model for this data, with $x = 0$ corresponding to 1980.

1. We first rewrite our table.

| | | | | | | |
|-------------------|------|------|--------|--------|--------|-------|
| Year | 0 | 5 | 10 | 15 | 20 | 23 |
| Enrollment | 9457 | 9479 | 10,845 | 11,092 | 11,750 | 11894 |

2. Using your calculator, you want to make a scatter plot the data in the table. However, you must first enter the data into two lists, one for x and one for y .

From the home page, hit [APPS]->6:Data/Matrix Editor->3:New. For Type, hit \odot ->3:List. For Folder, choose Main, and then for Variable, type in enroll [alpha], followed by [ENTER] (see Figure 1). Now hit [ENTER] again. Then type in the numbers for the years in column c1, each followed by [ENTER] or \odot . Then hit \odot , followed by \blacklozenge \odot to get to the top of the second column (see Figure 2). Then enter the six numbers for the enrollment similarly in column c2.



Figure 1

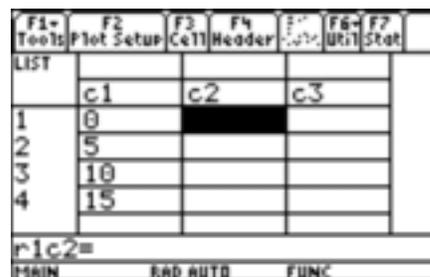


Figure 2

3. We plot our data.

Hit [F2] (Plot setup). Turn off any active plot that has a check in front of it by moving the cursor to the plot and hitting [F4] (\checkmark), and then move the cursor to an undefined plot and hit [F1] (Define). Choose Scatter for Type and Box for Mark. Type c1 for x (c [alpha] 1 should do it) and c2 ([alpha] c2) for y (See Figure 3). Then hit [ENTER] until you are back at the plot list. (Note that for regular function graphing, all plots should be off.)

Now hit \blacklozenge [Y=], and turn off any function that has a check in front of it by moving the cursor to the function and hitting [F4] (\checkmark). Then hit [F2] (Zoom)-> 9:ZoomData to get the scatter plot. Now hit \blacklozenge [WINDOW], and make xmin=-2, xmax=25, xscl=5, yminl=9000, ymax=12500,

$yscl=500$, and $xres=1$. Then hit \square [GRAPH] to get the scatter plot where each tic mark on the x-axis now represents a distance of 5 and each tic mark on the y-axis a distance of 500 (see Figure 4).

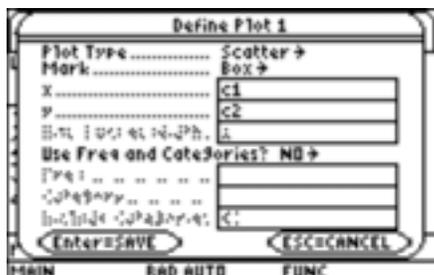


Figure 3

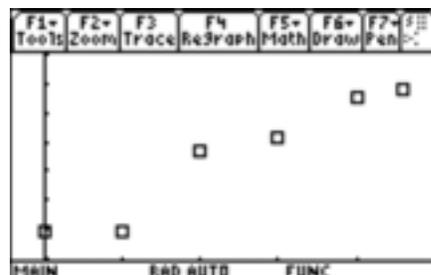


Figure 4

The points appear to be somewhat linear, so a **linear model** may be appropriate. But what line do we use? We know two points determine a line, so let's choose the first and last points (we could choose any pair of points). These points are (0, 9457) and (23,11894). The slope is 105.9565 to four decimal places, and we are given the y-intercept. The equation is $y=105.9565 x + 9457$.

4. We add the linear model of part 3 to the 6 data points on the graph in Figure 4.

Press \square [Y=] and then enter the linear function from part 3 as y_1 (for instance), pressing \square [ENTER] after it has been completely entered on the entry line. Then hit \square [GRAPH]. On the TI-89, your data points should have remained and the linear graph will now be plotted with it. Notice that the graph goes through the center of the boxes for the points (0, 9457) and (23,11894).

5. Our line goes through the first and last points, but misses the others by varying distances. The "best" linear model uses all the data points instead of two arbitrary ones as we did in part 3 along with a statistical technique called regression. Linear regression finds the unique line that minimizes the sum of the squares of the vertical distances from each of the data points to the line. The technique for computing this line is learned in Calculus III. We now compute the linear regression formula for our data.

Press \square [APPS]->6:Data/Matrix Editor->1:Current to return to the Data/Matrix Editor. Hit \square [F5](Calc), for Calculation Type choose linear regression by hitting \square [5]:LinReg. Enter c_1 for x and c_2 for y as was done earlier. For Store RegEQ to, choose $y_2(x)$ or any other function that is clear or you want to replace (See Figure 5). Then hit \square [ENTER]. The formula is of the form $y=a x + b$. Using the a and b given, we have that the linear regression equation is $y = 117.262687 x + 9326.137313$.

Notice also that we are given **corr = .969132**. This is the correlation coefficient, and is often abbreviated **r**. It is a statistical measure of how close the data is to lying on a line. The correlation coefficient takes on values between negative and positive 1, inclusively. It has the same sign as the slope of the regression line. The closer the absolute value of the correlation

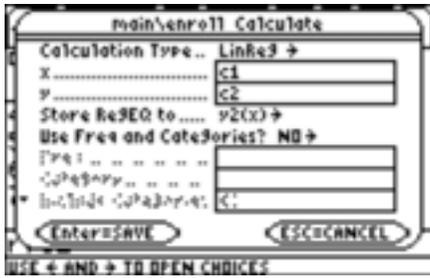


Figure 5



Figure 6

coefficient is to 1, the better the regression line fits the points. Perfect correlation (all the data points line on a single line – the regression line) has a correlation coefficient of ± 1 . Correlation coefficients near 0 indicate a poor fit.

6. Let's graph this **least-squares linear regression** model.

Press OK to clear the screen in Figure 6, then hit \blacklozenge [Y=], move the cursor onto y1, and press $\boxed{F4}$ (\surd) to deselect the first equation. Then hit \blacklozenge [GRAPH] to get the regression line with the data points. You can see a better fit than previously.

(b) If the model remains accurate in the future, estimate public college enrollment in 2006 and 2009.

There are two ways to do this.

The first way to do this is to use the Numeric Solver. Press $\boxed{\text{APPS}}$ ->9:Numeric Solver, hit $\boxed{\text{CLEAR}}$ to clear any previous equation. Assuming y2 is the equation used, enter y2(x) as the equation and press \ominus or $\boxed{\text{ENTER}}$ (See Figure 7 – exp=y2(x) and x may have different values). Then press \ominus or $\boxed{\text{ENTER}}$ until you get to x, type in your value for x (26 or 29), press \ominus to get to exp (which the display tells you equals y2(x)), and then press $\boxed{F2}$ (Solve) to get the corresponding y (see Figure 7). When finished, press $\boxed{\text{APPS}}$ ->6:Data/Matrix Editor->1:Current to return to the Data/Matrix Editor.

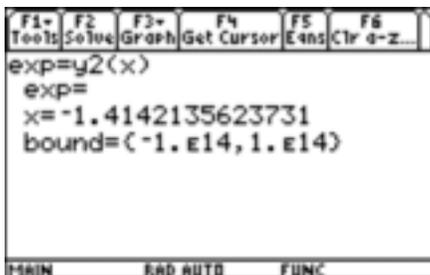


Figure 7

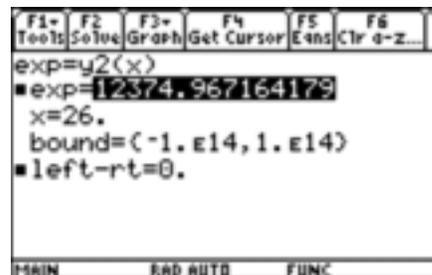


Figure 8

The second method is to hit $\boxed{\text{HOME}}$ and assuming y2 is the equation used, enter y2(26) on the entry line for 2006, and press $\boxed{\text{ENTER}}$. The corresponding y value then appears. Then hit

\rightarrow \rightarrow \leftarrow \leftarrow 29 to get $y_2(29)$ on the entry line for 2009, and press $\boxed{\text{ENTER}}$. The corresponding y value then appears.

(c) According to this model, when will public college enrollment reach 13,000,000.

Go back to the Numeric Solver by pressing $\boxed{\text{APPS}}$ ->9:Numeric Solver. Assuming y_2 is the equation used, refer back to Figure 7. Then press \downarrow or \leftarrow until you get to exp= , type in 13000, press \downarrow to get to x , and then press $\boxed{\text{F2}}$ (Solve) to get the corresponding x , which turns out to be 31.33. Thus an enrollment of 13,000,000 will be attained around May 1, 2111.