CHAPTER 1

Functions and Linear Models

1. Functions

**Definition.** A function is a rule that associates each input with exactly one output. □

<table>
<thead>
<tr>
<th>Millions of cameras made</th>
<th>Cost in million $</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>175</td>
</tr>
<tr>
<td>5</td>
<td>260</td>
</tr>
<tr>
<td>8</td>
<td>305</td>
</tr>
<tr>
<td>12</td>
<td>395</td>
</tr>
</tbody>
</table>

**Example.** (1)

For a particular model, the cost in millions of $ (output) is a function of the number of millions made (input).

(2) At any given time (input), this class room has exactly one temperature (output).

(3) If you make $10 an hour, your wage (output) is a function of the number of hours you work (input).

(4) The rule that associates a number (input) with the numbers it is a square of (output) is not a function. For example, 9 is the square of both 3 and −3. Uniqueness of output is lost. □
(5) Consider the following table, representing the number of diamonds in several collections of precious stones:

<table>
<thead>
<tr>
<th># of stones in collection</th>
<th># of diamonds in collection</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>15</td>
</tr>
<tr>
<td>20</td>
<td>7</td>
</tr>
<tr>
<td>28</td>
<td>14</td>
</tr>
<tr>
<td>33</td>
<td>21</td>
</tr>
</tbody>
</table>

This is not a function, since knowing you have 33 stones (input) does not indicate exactly how many diamonds you have (output), since you could have 15 or 21.

Do you have a function if you exchange the inputs and outputs? Why or why not?

(6) Same as above:

<table>
<thead>
<tr>
<th># of stones in collection</th>
<th># of diamonds in collection</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>12</td>
</tr>
<tr>
<td>36</td>
<td>12</td>
</tr>
<tr>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td>22</td>
<td>18</td>
</tr>
</tbody>
</table>

Here the number of diamonds (output) is a function of the number of stones (input). Several inputs can have the same output, just so each input gives a unique output.

Do you have a function if you exchange the inputs and outputs? Why or why not? □
**Function Notation**

Suppose a person makes $10.25 an hour. A partial table giving wages as a function of hours is:

<table>
<thead>
<tr>
<th>Hours</th>
<th>Wages in $</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>102.50</td>
</tr>
<tr>
<td>20</td>
<td>205.00</td>
</tr>
<tr>
<td>30</td>
<td>307.50</td>
</tr>
<tr>
<td>40</td>
<td>410.00</td>
</tr>
</tbody>
</table>

Suppose we let $h$ stand for the hours worked, and give the function the name $W$. We have

<table>
<thead>
<tr>
<th>$h$</th>
<th>$W$</th>
<th>$W(h)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>input</td>
<td>function</td>
<td>output</td>
</tr>
<tr>
<td>value</td>
<td>name</td>
<td>value</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>205.00</td>
</tr>
<tr>
<td>30</td>
<td></td>
<td>307.50</td>
</tr>
<tr>
<td>40</td>
<td></td>
<td>410.00</td>
</tr>
</tbody>
</table>

We say $W(h) = 10.25h$.

We call $h$, representing the input value the **independent variable**.

We call $W(h)$, representing the output value the **dependent variable** since its value depends on the choice of input variable.

$$W(15.5) = 10.25 \times 15.5 = 158.875 \approx 158.88$$
$$W(53) = 10.25 \times 53 = 543.25$$

**Note.** The generic independent variable is $x$, the generic function name is $f$, and the generic dependent variable is $y$, so $y = f(x)$. 
Example. Suppose $f(x) = x^2 + 3x + 4$.

$$f(5) = 5^2 + 3 \cdot 5 + 4 = 25 + 15 + 4 = 44$$

$$f(-3) = (-3)^2 + 3 \cdot (-3) + 4 = 9 - 9 + 4 = 4$$

$$f(\pi) = \pi^2 + 3\pi + 4$$

$$f(h + 2) = (h + 2)^2 + 3(h + 2) + 4 = h^2 + 4h + 4 + 3h + 6 + 4 = h^2 + 7h + 14$$

$$f(-x) = (-x)^2 + 3(-x) + 4 = x^2 - 3x + 4$$

$$f(x^3) = (x^3)^2 + 3(x^3) + 4 = x^6 + 3x^3 + 4$$

Graphs of Functions

No matter which variables you are using, the independent variable goes with the horizontal (usual $x$-) axis and the dependent variable goes with the vertical (usual $y$-) axis. In fact, your calculator is limited to $x$ and $y$ for graphing.

Example. You have a function $C(h) = 3h^2 + 2$. For calculator use, you translate to $y = 3x^2 + 2$. □
Example. Given the graph below, which is part of the graph of a function. Assume $y = f(x)$.

\[ f(-3) = 1 \]
\[ f(-1) = 3 \]
\[ f(1) = 2 \]
\[ f(0) \approx 2.2 \]

How can you tell if a graph is the graph of a function?
**Vertical Line Test.** If every vertical line drawn on a graph intersects the graph in at most one point, then the graph is the graph of a function. Otherwise the graph is not the graph of a function.

Some functions are **discontinuous**, i.e., they have a break in them.

**Example.** $g(x) = \begin{cases} -x, & x < 0 \\ x + 1, & x \geq 0 \end{cases}$ is a function.

The open dot at $(0, 0)$ indicates that $(0, 0)$ is not a point of the graph, while the closed dot indicates $(0, 1)$ is part of the graph.
Graphing with the TI

**EXAMPLE.** Graph \( f(x) = x^3 - 3x - 1 \).

Press \([Y=]\).  
Clear any current formulas and type \( Y_1 = X \triangle 3 - 3X - 1 \).

Use **ZOOM\6:ZSTANDARD.**  
Use Use **TRACE** – you get funny \( x \)-values.

Use **ZOOM\4:ZDECIMAL.**  
You get a window of \([-4.7, 4.7] \times [-3.1, 3.1] \) or \(-4.7 \leq x \leq 4.7 \) and \(-3.1 \leq y \leq 3.1 \).

Use multiples of 4.7 for \( X\text{min} \) and \( X\text{max} \).  
Trace gaps are 0.1 for 4.7, 0.2 for 9.4, 0.3 for 14.1, etc. Use 9.4.

Then use **ZOOM\0:ZoomFit.**  
We get good tracing, but this is not a good window since important aspects of the graph get lost.

A good window here is \([-4.7, 4.7] \times [-10, 10] \).

Hit \( 2\text{nd}\ Calc\1:VALUE. \) Enter \( X=-2 \) and hit **ENTER.** You get \( y=-3 \), so \( f(-2) = 3 \). Hit 3 **ENTER** to get \( f(3) = 17 \). For \( X \), you can enter any number between \( X\text{min} \) and \( X\text{max} \).

Enter \( X=0 \) to find the \( y \)-intercept. The \( y \)-intercept is \(-1 \).

There are 3 \( x \)-intercepts. We see that the middle one is between \(-1 \) and \( 0 \) and none of the others are between those numbers.

Hit \( 2\text{nd}\ Calc\2:ZERO. \) Use \(-1 \) **ENTER** for **Left bound, \( 0 \)** **ENTER** for **Right bound, \( \)**, and just **ENTER** for **Guess.**  
This \( x \)-intercept is \(-.3472964 \).

To make a table of values for every 0.5 starting at \(-10\):

1. Hit \( 2\text{nd}\ Tblset. \)
2. Hit \(-10 \) **ENTER** for **Tblstart.**
3. Hit \(.5 \) **ENTER** for **\( \Delta Tbl. \)**
4. Now hit \( 2\text{nd}\ Table. \)
Domain and Range

DEFINITION. The set of all possible values of the independent variable of a function is called the domain. The set of all possible values of the dependent variable of a function is called the range.

NOTE. Two things completely determine a function: the rule and the domain. Change either and you change the function.

Unless otherwise specified, the domain of a function is all real numbers for which the rule gives a real number.

There are 3 common situations in which the domain is restricted to a subset of the real numbers.

(1) A zero in the denominator.

EXAMPLE. \( g(t) = \frac{3t + 5}{t^2 - 4} \).

Our domain cannot contain values of \( t \) that make the denominator 0. So we solve:

\[
\begin{align*}
t^2 - 4 &= 0 \\
(t + 2)(t - 2) &= 0 \\
t + 2 &= 0 \text{ or } t - 2 = 0 \\
t &= -2 \text{ or } t = 2
\end{align*}
\]

The domain is all real numbers except 2 and \(-2\). \( \square \)
(2) A negative value under a square root symbol (radical).

**Example.** \[ f(x) = \frac{\sqrt{x + 3}}{x - 7} \]

For \( \sqrt{x + 3} \) to be a real number,
\[
x + 3 \geq 0 \implies x \geq -3
\]
So no number less than \(-3\) can be in the domain.
Also, \( x - 7 \) cannot be 0. Why?
\[
x - 7 = 0
\]
\[
x = 7
\]
Thus the domain consists of all real numbers greater than or equal to \(-3\), except for 7.

(3) The context of a word problem

**Example.**
\[
n = \text{number of a certain model of car}
\]
\[
W(n) = \text{total cost of the cars}
\]
Here, the domain would have to be the nonnegative integers or whole numbers.
2. Linear Functions

**Definition.** A linear function is a function whose graph is a line.

The line passing through any two points \((x_1, y_1)\) and \((x_2, y_2)\) with \(x_1 \neq x_2\) is referred to as the graph of a linear function.

A linear function has a constant rate of change. Increasing the domain value by one unit will always change the corresponding range value by the same amount.

**Definition.** The slope of a linear function (and of a line) is the change in output that occurs when the input is increased by one unit. The slope \(m\) may be calculated by dividing the difference of any two outputs by the difference in the corresponding inputs. That is,

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

where \((x_1, y_1)\) and \((x_2, y_2)\) are data points of the linear function (or points on the line).
2. LINEAR FUNCTIONS

Example. (1) (8, 4) and (4, -6) are data points.

\[ m = \frac{4 - (-6)}{8 - 4} = \frac{10}{4} = \frac{5}{2} \]

(2) (-6, -2) and (3, -5) are data points.

\[ m = \frac{-5 - (-2)}{3 - (-6)} = \frac{-3}{9} = -\frac{1}{3} \]
(3) (3, -2) and (8, -2) are data points.

\[ m = \frac{-2 - (-2)}{8 - 3} = \frac{0}{5} = 0 \]

(4) (5, 7) and (5, 3) are data points.

\[ m = \frac{7 - 3}{5 - 5} = \frac{4}{0} = \text{undefined} \]

We say the slope is undefined or there is no slope.

**Definition.** The y-intercept is the point on the graph where the function intersects the y-axis. It occurs when the value of the independent variable (usually \( x \)) is 0. It is formally written as the ordered pair \((0, b)\), but \(b\) itself is often called the y-intercept.
The $x$-intercept is the point on the graph where the function intersects the $x$-axis. It occurs when the value of the dependent variable (usually $y$) is 0. It is formally written as the ordered pair $(a, 0)$, but $a$ itself is often called the $x$-intercept.

**Example.** Find the intercepts of $y = 4x + 7$.

To find the $y$-intercept, set $x = 0$:

\[ y = 4 \cdot 0 + 7 \]
\[ y = 7 \]

The $y$-intercept is $(0, 7)$ or just 7.

To find the $x$-intercept, set $y = 0$:

\[ 0 = 4x + 7 \]
\[ -4x = 7 \]
\[ x = -\frac{7}{4} \]

The $x$-intercept is $\left(-\frac{7}{4}, 0\right)$ or just $-\frac{7}{4}$.

\[
m = \frac{7 - 0}{0 - (-\frac{7}{4})} = \frac{7}{\frac{7}{4}} = 7 \cdot \frac{4}{7} = 4. \]

On the TI:

(1) Press Y=, clear any functions, and enter \( Y_1=4X+7 \).

Press ZOOM/6:ZSTANDARD.

Find intercepts as described earlier.

To find the slope: 2nd Calc/6:dy/dx/ENTER.

(2) Hit Math/0:Solver.

Hit ↑ until “EQUATION SOLVER” appears.

Hit CLEAR.

Enter eqn:0=y-4x-7, then ENTER. In entering an equation such as “left=right,” you enter it as eqn:0=left-right or eqn:0=right-left.

Put Y=0 or Y=any other #, then ↓.

Then hit ALPHA/SOLVE to get \( X=1.75 \) as X-intercept (or X-value for the Y-value you entered). The point given by \( (X,Y) \) is a point on the line.

Now put X=0 or X=any other #, then ↑.

Then hit ALPHA/SOLVE to get \( y=7 \) as Y-intercept (or Y-value for the X-value you entered). The point given by \( (X,Y) \) is a point on the line.

This method can be used to generate a table of points for a linear function.
Linear Equations
The graph of any line may be represented by a linear equation.
The equation of a \textit{vertical line} passing through a point \((a, b)\) is \(x = a\).

The equation of a \textit{horizontal line} passing through a point \((a, b)\) is \(y = b\).
Slope-intercept form of a line

A linear function with slope $m$ and $y$-intercept $(0, b)$ has the equation

$$y = mx + b.$$ 

**Problem** (page 28 #38).

Let $x = \#$ of apples eaten

Let $F(x) = \#$ of grams of dietary fiber

Since we know one banana is eaten, one consumes 3.3 grams of fiber from the one banana. thus the $y$-intercept is $(0, 3.3)$. Why?

Each apple gives 5.7 grams of fiber, so the slope is 5.7. Why? Then

$$F(x) = 5.7x + 3.3.$$ 

To get 30 grams of fiber,

$$5.7x + 3.3 = 30$$

$$5.7x = 26.7$$

$$x = \frac{26.7}{5.7} \approx 4.68$$

thus you would have to eat 5 apples. □
PROBLEM (page 28 #34).

Since

\[
\begin{align*}
79.80 - 59.85 &= 19.95, \\
99.75 - 79.80 &= 19.95, \\
119.70 - 99.75 &= 19.95,
\end{align*}
\]

The total cost increases by the same amount each time a person is added to the group.

This amount, 19.95, is the slope of a linear function. So, if

- \( x = \# \) of people in the group,
- \( y = \) total cost of admission for the group,

\[
y = 19.95x + b.
\]

Put in any data point, such as \((3, 59.85)\), in for \((x, y)\) and solve for \(b\):

\[
\begin{align*}
59.85 &= 19.95(3) + b \\
59.85 &= 59.85 + b \\
0 &= b
\end{align*}
\]

Our function is \(y = 19.95x\) \(\square\)

To find the slope-intercept form of a line from two points:

(1) Find the slope.

(2) Substitute the slope for \(m\) in \(y = mx + b\).

(3) Choose one point and substitute the output for \(y\) and the input for \(x\).

(4) Solve for \(b\).

(5) Substitute this number for \(b\) in \(y = mx + b\).
Example. Find the equation of the line passing through the points \((-2, -6)\) and \((3, 4)\).

Solution

\[ y = mx + b \]

\[ m = \frac{4 - (-6)}{3 - (-2)} = \frac{10}{5} = 2 \]

\[ y = 2x + b \]

Substitute either point for \(x\) and \(y\), and then solve for \(b\).

\[ 4 = 2(3) + b \]

\[ 4 = 6 + b \]

\[ -2 = b \]

The equation is \(y = 2x - 2\). \(\square\)

**Standard form of a line**

Any linear equation may be written as

\[ ax + by = c \]

where \(a\), \(b\), and \(c\) are real numbers, \(a\) and \(b\) not both 0.

If \(a = 0\), the graph is a horizontal line.

**Example.**

\[ 3y = 9 \text{ or } y = 3 \]

If \(b = 0\), the graph is a vertical line.

**Example.**

\[ 2x = 8 \text{ or } x = 4 \]

Unless \(c = 0\), the easiest way to graph a linear equation in standard form is to find the intercepts, which are easy to find in this case.

The \(x\)-intercept is \(\frac{c}{a}\) and the \(y\)-intercept is \(\frac{c}{b}\).
Example. \( 4x - 5y = 20 \).

\[ x\text{-intercept: } x = \frac{20}{4} = 5. \]

\[ y\text{-intercept: } y = \frac{20}{-5} = -4. \]

Compare this to slope-intercept graphing:

\[ -5y = -4x + 20 \]

\[ y = \frac{4}{5}x - 4 \]

Then start at \(-4\) on the \(y\)-axis for a first point, the go 5 to the right and 4 up for a second point. \(\square\)
Point-slope form of a line

A linear function written as

\[ y - y_1 = m(x - x_1) \]

has slope \( m \) and passes through the point \((x_1, y_1)\).

**Example.** Find the line passing through the points \((2, 3)\) and \((5, 7)\).

**Solution**

Find the slope: \( m = \frac{7 - 3}{5 - 2} = \frac{4}{3} \).

Then use either point with the point-slope form:

\[
\begin{align*}
    y - 3 &= \frac{4}{3}(x - 2) \\
    y - 3 &= \frac{4}{3}x - \frac{8}{3} \\
    y &= \frac{4}{3}x + \frac{1}{3} \\
    -\frac{4}{3}x + y &= \frac{1}{3} \\
    -4x + 3y &= 1 \\
    4x - 3y &= -1
\end{align*}
\]

\(\square\)
EXAMPLE. Graph the linear function $y = 3x - 7$.

Solution

The $y$-intercept is $-7$, so $(0, -7)$ is a point on the line. Only one more point is needed. If $x = 3$, $y = 3(3) - 7 = 2$, so $(3, 2)$ is a second point on the line. Of course, we could use the slope of $3 = \frac{3}{1}$ to find a second point also.

![Graph of linear function]

PROBLEM (page 29 #44).

If $f(x) = mx + b$ and $g(x) = nx + c$ are parallel (do not intersect), what can you say about $m$, $b$, $n$, and $c$?

Solution

$b \neq c$, or else the lines would intersect.

The slopes must be the same, so $m = n$. 
3. Linear Models

We consider linear, near linear, and piecewise linear relationships.

**Definition.** Two quantities are said to be directly proportional if the ratio of the output to the input is a constant.

We have

\[ \frac{y}{x} = k, \]

where \( k \) is the constant of proportionality.

Thus quantities which are directly proportional can be modeled by the linear equation

\[ y = kx. \]

**Problem** (page 47 #12).

(a) \( x = \# \) of large orders of French fries

\[ y = \# \text{ of fat grams} \]

\[ y = 26x, \] a proportion.

(b) \( x = \# \) of Big Macs

\[ y = 34x, \] a proportion.

(c) 2

(d) 1

(e) \( x = \# \) of combination meals

\[ y = 60x, \] a proportion.

You can eat one combination meal. □
Determining if a data set is directly proportional

**Example.** This is Example 2 from page 30 redone with different data – buying gas at the pump.

<table>
<thead>
<tr>
<th>Gallons</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.50</td>
<td>10.15</td>
</tr>
<tr>
<td>6.28</td>
<td>18.21</td>
</tr>
<tr>
<td>17.34</td>
<td>50.28</td>
</tr>
</tbody>
</table>

We check to see if this relationship is directly proportional:

\[
\frac{10.15}{3.50} = 2.9, \quad \frac{18.21}{6.28} = 2.89968, \quad \frac{50.28}{17.34} = 2.89965
\]

As given, the data is not directly proportional since the ratios are not equal, although they are very close.

But experience tells us that the cost of gas is directly proportional to the amount purchased. The issue is that gas is priced to the tenth of a cent.

From our data, suppose we use the directly proportional model \( y = 2.899x \).

<table>
<thead>
<tr>
<th>( x = \text{Gallons} )</th>
<th>Cost</th>
<th>( y = \text{cost by model estimate} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.50</td>
<td>10.15</td>
<td>10.14965</td>
</tr>
<tr>
<td>6.28</td>
<td>18.21</td>
<td>18.211372</td>
</tr>
<tr>
<td>17.34</td>
<td>50.28</td>
<td>50.284266</td>
</tr>
</tbody>
</table>

The cost you pay is the cost from the model rounded to the nearest cent. \( \square \)
Linear Regression

**Example.** Enrollment in public colleges (in thousands) in selected years is shown in the table below:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>enrollment ((y))</td>
<td>9457</td>
<td>9479</td>
<td>10,845</td>
<td>11,092</td>
<td>11,750</td>
<td>11,894</td>
</tr>
</tbody>
</table>

A quick check with \(x = \) year and \(y = \) enrollment shows that the data is not directly proportional. We plot the data using the calculator, but first, to simplify, we change the data so that \(x = \) years since 1980, so that \(x = 0\) in 1980. This is an often used technique.

<table>
<thead>
<tr>
<th>year ((x))</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>23</th>
</tr>
</thead>
<tbody>
<tr>
<td>enrollment ((y))</td>
<td>9457</td>
<td>9479</td>
<td>10,845</td>
<td>11,092</td>
<td>11,750</td>
<td>11,894</td>
</tr>
</tbody>
</table>

We create a scatterplot by

1. following steps 1–6 in the Technology Tip on page 33;
2. pressing Y= and clearing any functions
3. following steps 1–4 in the Technology Tip on page 37

We notice that, while the points are not in a straight line, they are “close” to being linear. We need to adjust our graph since the point on the \(y\)-axis is almost hidden and \(Xscl=1\) and \(Yscl=1\) give inappropriate tic marks. Press WINDOW and enter \(Xmin=-2, Xmax=25, Xscl=5, Ymin=9000, Ymax=12500,\) and \(Yscl=500.\) Then press GRAPH.
Now the $x$-axis goes from 0–25 by 5’s and the $y$-axis goes from 9000–12500 by 500’s. **TRACE** through the points.

The points are close to a line, but which line? You could try any pair of points with the point-slope formula to get a line that goes through at least 2 of the points, but there is a standard better way.

The “best” linear model uses all the data points instead of two arbitrary ones and is called (linear) regression.

Linear regression uses calculus to find the line that minimizes (makes the smallest) the sum of the squares of the vertical distances from each point to the chosen line.

Your calculator can do this by following steps 7–9 in the Technology Tip on page 34 with two adjustments.

1. Before step 7, go to **Y=** and clear any functions, but leave **Plot1** on.
2. After choosing **4:LinReg(ax+b)**, press **VARS**, then → and **ENTER ENTER ENTER** to put the result into **Y1** under **Y=**.
The regression equation of best fit is
\[ y = 117.2626866x + 9326.137313. \]

The number \( r = .9691320811 \) is called the correlation coefficient. This is a number between \(-1\) and \(1\) that measures the degree to which two variables are linearly related.

\( r > 0 \) when the regression equation has a positive slope and \( r < 0 \) when the regression equation has a negative slope.

If \( r = \pm 1 \), the points lie on a line, The more closely the variables are related, the closer \( r \) is to \( \pm 1 \). \( r = 0 \) means the variables are not linearly related.

The number \( r^2 = .9392169905 \) is the coefficient of determination. The closer this is to \( 1 \), the better the line models the data.

To graph the regression equation, follow steps 1–6 in the Technology Tip on page 35 or just hit GRAPH. We get the following graph:
Although the line misses literally every point, it minimizes the sum of the squares of the errors at each point.

We can use the line (or regression equation) to estimate the enrollment $y$ at in-between years $x$.

Press 2nd Calc/1:Value/12 to get an enrollment of 10733.29 in 1992, or

$$y(12) = 10733.29.$$ 

To see what the model would predict in 2010 if it continued to hold, put 30 into the formula for $x$ and then find $y$, or, change $\textbf{Xmax}$ to 30 in WINDOW, go back to GRAPh, and press 2nd Calc/1:Value/30 to get

$$y(30) = 12844.018.$$ 

What does it mean that the slope of the regression equation is roughly 117?

It means that the public college enrollment is growing by about 117,000 per year. This is important data for strategic planning. 

Piecewise Linear Models

A function of the form

$$f(x) = \begin{cases} 
ax + b & x \leq c \\
qx + g & x > c 
\end{cases}$$

is called a piecewise linear function.

**Example.** The absolute value function

$$f(x) = |x| = \begin{cases} 
x & x \geq 0 \\
-x & x < 0 
\end{cases}.$$
TI: \( Y_1 = \text{MATH/NUM/ABS}(X) \) to get \( Y_1 = \text{ABS}(X) \)

or

\[ Y_2 = X \cdot (X \text{ 2nd TEST} \geq 0) + (-X \cdot (X \text{ 2nd TEST} < 0)) \]

\[ \text{to get } Y_2 = X \cdot (X \geq 0) + (-X) \cdot (X < 0). \quad \square \]

**Problem** (page 47 #8). A cell phone plan costs $29.99 per month for 150 minutes, with additional time at $0.35 per minute or portion of a minute.

Let \( t = \# \) of minutes used per month

\( C(t) = \text{cost for } t \text{ minutes} \)

For \( t \leq 150, \]

\[ C(t) = 29.99 \]

For \( t > 150, \]

\[ C(t) = 29.99 + .35(\text{extra minutes}) \]
\[ = 29.99 + .35(t - 150) \]
\[ = 29.99 + .35t - 52.50 \]
\[ = .35t - 22.51 \]

Thus \( C(t) = \begin{cases} 29.99 & 0 \leq x \leq 150 \\ .35t - 22.51 & t > 150 \end{cases} \)

\[ \text{TI: } Y_1 = 29.99(\leq 150) + (.35x - 22.51)(x > 150) \]

**Window**: \([0, 400] \times [0.120]\)