Functions of Two Variables

```
> restart:with(plots):with(DEtools):
```

The graph of a function $z = F(x, y)$ is a surface in three-dimensional space. We graph the function
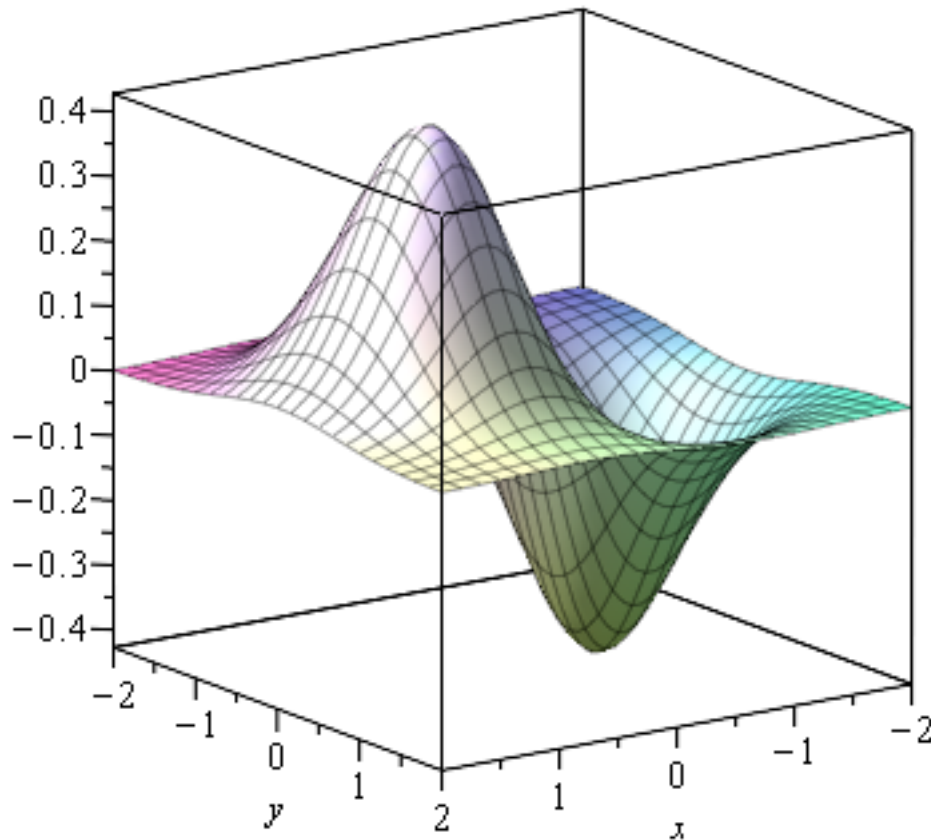$$z = F(x, y) = x\,e^{-x2\,-\,y2}.$$

```
> z:=x*exp(-x^2-y^2);
```

$$z := x\,e^{-x^2\,-\,y^2}$$

We look at a plot of this graph using plot3d.

```
> plot3d( z, x=-2..2, y=-2..2, axes=BOXED, title=`A Surface Plot`);
```
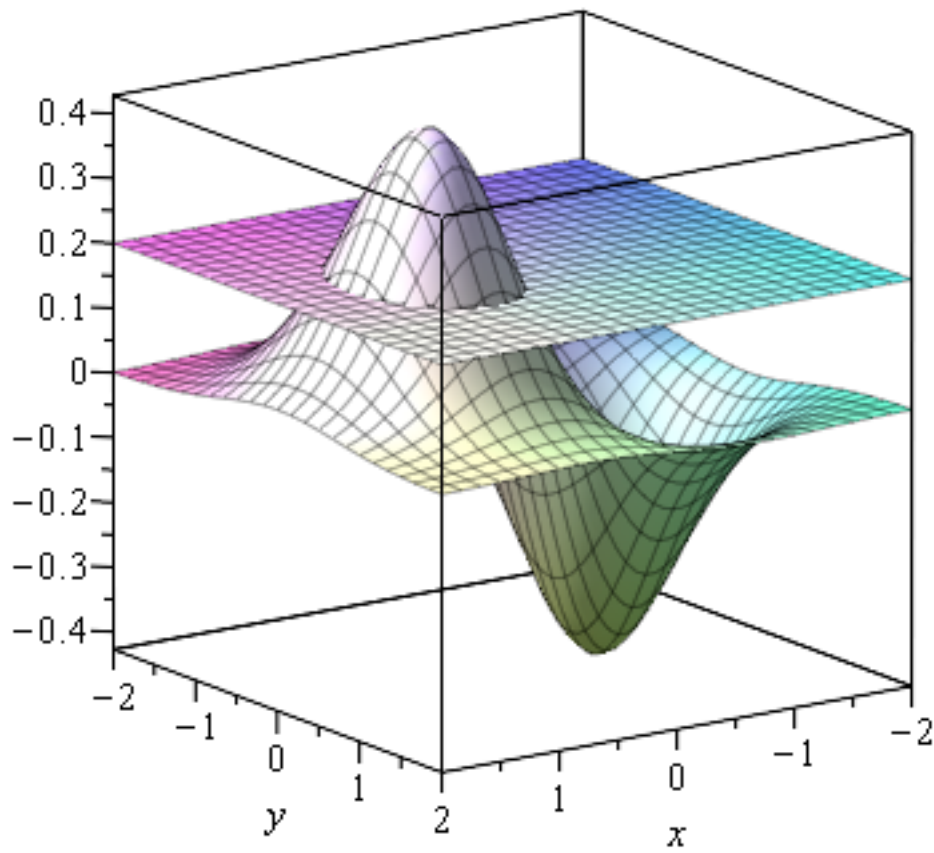
A Surface Plot
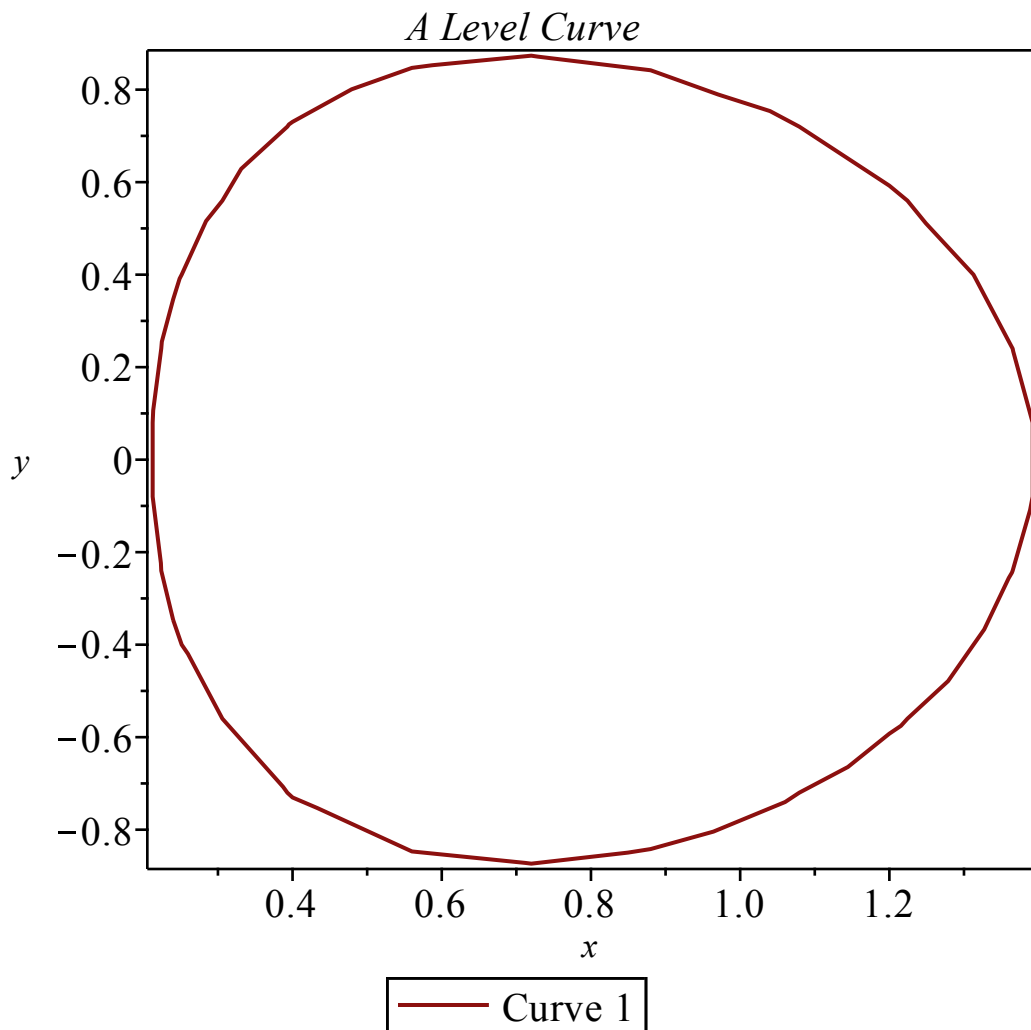

We cut the surface by a horizontal plane $z = 0.2$.

```
> plot3d( {z,0.2}, x=-2..2, y=-2..2, axes=BOXED,title=`A Surface Plot
  Cut by a Plane`);
```

1

A Surface Plot Cut by a Plane

We project the intersection of the plane with the surface onto the *x-y* plane to get the level curve $z = F(x, y) = x\,e^{-x^2 - y^2} = 0.2$.
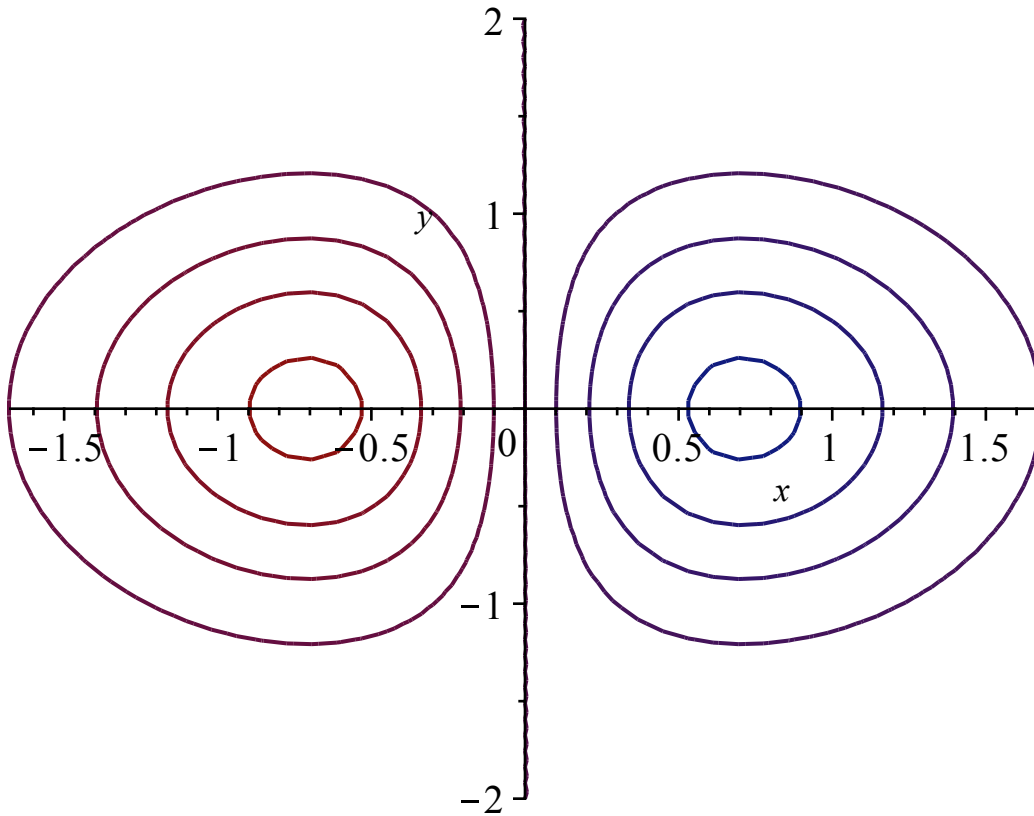
```
> implicitplot( {z=0.2}, x=-2..2, y=-2..2, axes=BOXED,
  title=`A Level Curve`);
```

A Level Curve

We can do several level curves at once by doing a contourplot. For each C in the contour list, the level curve $z = F(x, y) = C$ is plotted.

```
> contourplot( {z}, x=-2..2, y=-2..2, title=`A Contour Plot`,
  contours=[-.4,-.3,-.2,-.1,0,.1,.2,.3,.4]);
```

A Contour Plot

Curve 1	Curve 2	Curve 3	Curve 4
Curve 5	Curve 6	Curve 7	Curve 8
Curve 9			

Note that the bluer the contour, the greater the value (of C), and the redder the contour, the lesser the value (of C). Thus, here, positive contours are to the right of the y-axis, the negative contours to the left.