Parameterized Curves

```maple
restart:with(plots):with(VectorCalculus):
setoptions3d(axes=NORMAL,labels=["x","y","z"],orientation=[20,70]):

We first graph the curve parameterized by $x = 8 \cos(t) + 2 \cos(7t), y = 8 \sin(t) + 2 \sin(7t)$. But we
graph the curve in eight stages, noting $t=0..2\pi$ produces the entire curve.

```maple
> p1:=plot([[8*cos(t)+2*cos(7*t),8*sin(t)+2*sin(7*t),t=0..Pi/4],
scaling=constrained,color=red):
p2:=plot([[8*cos(t)+2*cos(7*t),8*sin(t)+2*sin(7*t),t=Pi/4..Pi/2],
scaling=constrained,color=green):
p3:=plot([[8*cos(t)+2*cos(7*t),8*sin(t)+2*sin(7*t),t=Pi/2..3*Pi/4],
scale=constrained,color=blue):
p4:=plot([[8*cos(t)+2*cos(7*t),8*sin(t)+2*sin(7*t),t=3*Pi/4..Pi],
scale=constrained,color=orange):
p5:=plot([[8*cos(t)+2*cos(7*t),8*sin(t)+2*sin(7*t),t=Pi..5*Pi/4],
scale=constrained,color=violet):
p6:=plot([[8*cos(t)+2*cos(7*t),8*sin(t)+2*sin(7*t),t=5*Pi/4..3*Pi/2],
scale=constrained,color=red):
p7:=plot([[8*cos(t)+2*cos(7*t),8*sin(t)+2*sin(7*t),t=3*Pi/2..7*Pi/4],
scale=constrained,color=blue):
p8:=plot([[8*cos(t)+2*cos(7*t),8*sin(t)+2*sin(7*t),t=7*Pi/4..2*Pi],
scale=constrained,color=red]):
display(p1,p2,p3,p4,p5,p6,p7,p8);
```
It is clear that the curve is being plotted in a counterclockwise direction. Let's find the length of this curve.

\[ f := 8 \cos(t) + 2 \cos(7t) : \]
\[ g := 8 \sin(t) + 2 \sin(7t) : \]
\[ s := \int \sqrt{\left( \frac{df}{dt} \right)^2 + \left( \frac{dg}{dt} \right)^2} \, dt, t = 0 \ldots 2\pi; \]
\[ s_{\text{approx}} := \text{evalf}(s); \]

\[ s := 24 \text{ EllipticE} \left( \frac{4}{3} \sqrt{7} \right) \]
\[ s_{\text{approx}} := 95.30571353 - 0.1 \]

Most of the integrals that we get for finding arc length are very difficult (or impossible) to evaluate using the Fundamental Theorem of Calculus. So we turn to a numerical method such as Simpson's rule. We can also find the arc length by using the \texttt{ArcLength} command from the \texttt{VectorCalculus} package.

\[ s_{\text{approx}} := \text{ArcLength}(<f, g>, t = 0 \ldots 2\pi); \]

\[ s_{\text{approx}} := 24 \text{ EllipticE} \left( \frac{4}{3} \sqrt{7} \right) \]

Now we move to three dimensions and plot the curve given parametrically by the equations \( x = \sin(t), \)
\( y = -\csc(t), \) \( z = \cot. \) We again break the formation of the plot into parts, but from \( \ell = \frac{\pi}{100} \ldots 99 \frac{\pi}{100}. \)
Why not \( t = 0 \ldots \pi \) you might ask?

> p1:=spacecurve([sin(t),-csc(t),cot(t)],t=Pi/100..Pi/4,color=red):
p2:=spacecurve([sin(t),-csc(t),cot(t)],t=Pi/4..Pi/2,color=green):
p3:=spacecurve([sin(t),-csc(t),cot(t)],t=Pi/2..3*Pi/4,color=blue):
p4:=spacecurve([sin(t),-csc(t),cot(t)],t=3*Pi/4..99*Pi/100,color=gold):
display(p1,p2,p3,p4);

Let's find the length of this curve.

> f:=sin(t):
g:=-csc(t):
h:=cot(t):
s:=evalf(ArcLength([f,g,h],t=Pi/100..99*Pi/100));

\[
s = 88.98543743
\]

Next we look at the intersection of the cone \( z = \sqrt{x^2 + y^2} \) with the plane \( y + 2z = 2 \).

> p1:=plot3d(sqrt(x^2+y^2),x=-3..3,y=-3..3,style=patchnogrid):
p2:=plot3d(1/2*(2-y),x=-3..3,y=-3..3,style=patchnogrid):
display(p1,p2);
We equate the two expressions for $z$ and solve for $y$ in terms of $x$.

\[ y := \text{solve}(\sqrt{x^2 + y^2} = \frac{1}{2}(2 - y), y); \]
\[ y := -\frac{2}{3} - \frac{2}{3}\sqrt{4 - 3x^2}, \quad -\frac{2}{3} + \frac{2}{3}\sqrt{4 - 3x^2} \]

We solve for $z$ in terms of $y$.

\[ z[1] := \frac{1}{2}(2 - y[1]); z[2] := \frac{1}{2}(2 - y[2]); \]
\[ z_1 := \frac{4}{3} + \frac{1}{3}\sqrt{4 - 3x^2}; \]
\[ z_2 := \frac{4}{3} - \frac{1}{3}\sqrt{4 - 3x^2} \]

We add the two curves which together form an ellipse.

\[ p3 := \text{spacecurve}([t, -2/3 - (2/3)\sqrt{4 - 3t^2}, 4/3 + (1/3)\sqrt{4 - 3t^2}], t = -2/\sqrt{3}..2/\sqrt{3}, \text{color=black}, \text{thickness=2}); \]
\[ p4 := \text{spacecurve}([t, -2/3 + (2/3)\sqrt{4 - 3t^2}, 4/3 - (1/3)\sqrt{4 - 3t^2}], t = -2/\sqrt{3}..2/\sqrt{3}, \text{color=black}, \text{thickness=2}); \]
\[ \text{display}(p1, p2, p3, p4); \]
Finally, we find the length of the curve of intersection.

```plaintext
> f1:=t:
g1:=-2/3-(2/3)*sqrt(4-3*t^2):
h1:=4/3+(1/3)*sqrt(4-3*t^2):
f2:=t:
g2:=-2/3+(2/3)*sqrt(4-3*t^2):
h2:=4/3-(1/3)*sqrt(4-3*t^2):
> s:=ArcLength(<f1,g1,h1>,t=-2/sqrt(3)..2/sqrt(3))+ArcLength(<f2,
g2,h2>,t=-2/sqrt(3)..2/sqrt(3));
> s:=evalf(s);

s := \frac{8}{3} \sqrt{3} \text{EllipticE}\left(\frac{1}{3}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}\right)

s := 8.344362537 \sim 0.1
```