Cyclic Groups

```
> restart;with(GroupTheory);
```

[<|> , *AbelianInvariants*, *AllPerfectGroups*, *AllSmallGroups*, *AllTransitiveGroups*, *Alt*, *AlternatingGroup*, *AreConjugate*, *AreIsomorphic*, *BabyMonster*, *CayleyGraph*, *CayleyTable*, *CayleyTableGroup*, *Center*, *Centraliser*, *Centralizer*, *Centre*, *ComplexProduct*, *ConjugacyClass*, *ConjugacyClasses*, *Conjugator*, *ConwayGroup*, *Core*, *CustomGroup*, *CycleIndexPolynomial*, *CyclicGroup*, *Degree*, *DerivedLength*, *DerivedSeries*, *DerivedSubgroup*, *DicyclicGroup*, *DihedralGroup*, *DirectProduct*, *DrawCayleyTable*, *DrawSubgroupLattice*, *ElementOrder*, *ElementaryGroup*, *Elements*, *Embedding*, *ExceptionalGroup*, *Exponent*, *FPGroup*, *Factor*, *FischerGroup*, *FittingSubgroup*, *FrattiniSubgroup*, *FreeGroup*, *GL*, *GaloisGroup*, *GeneralLinearGroup*, *GeneralOrthogonalGroup*, *GeneralUnitaryGroup*, *Generators*, *Group*, *GroupOrder*, *HaradaNortonGroup*, *HeldGroup*, *HigmanSimsGroup*, *Hypercenter*, *Hypercentre*, *IdentifySmallGroup*, *Index*, *Intersection*, *IsAbelian*, *IsAlternating*, *IsCommutative*, *IsCyclic*, *IsElementary*, *IsFinite*, *IsNilpotent*, *IsNormal*, *IsPerfect*, *IsPrimitive*, *IsRegular*, *IsSimple*, *IsSoluble*, *IsSolvable*, *IsSubgroup*, *IsSymmetric*, *IsTransitive*, *JankoGroup*, *Labels*, *LeftCoset*, *LeftCosets*, *LowerCentralSeries*, *LyonsGroup*, *MathieuGroup*, *McLaughlinGroup*, *MetacyclicGroup*, *Monster*, *NilpotencyClass*, *NilpotentResidual*, *NonRedundantGenerators*, *NormalClosure*, *Normaliser*, *NormaliserSubgroup*, *NormalizerSubgroup*, *NumGroups*, *NumPerfectGroups*, *NumTransitiveGroups*, *ONanGroup*, *Operations*, *Orbit*, *Orbits*, *OrthogonalGroup*, *PCore*, *PGL*, *PGU*, *PSL*, *PSU*, *PSp*, *PerfectGroup*, *PermApply*, *PermCommutator*, *PermConjugate*, *PermCycleType*, *PermDegree*, *PermFixed*, *PermInverse*, *PermLeftQuotient*, *PermOrder*, *PermParity*, *PermPower*, *PermProduct*, *PermRightQuotient*, *PermSupport*, *PermutationGroup*, *PresentationComplexity*, *ProjectiveGeneralLinearGroup*, *ProjectiveGeneralUnitaryGroup*, *ProjectiveSpecialLinearGroup*, *ProjectiveSpecialUnitaryGroup*, *ProjectiveSymplecticGroup*, *QuaternionGroup*, *RandomElement*, *Relators*, *RightCoset*, *RightCosets*, *RubiksCubeGroup*, *RudvalisGroup*, *SL*, *SearchSmallGroups*, *SearchTransitiveGroups*, *Simplify*, *SmallGroup*, *SolubleResidual*, *SolvableResidual*, *SpecialLinearGroup*, *SpecialOrthogonalGroup*, *SpecialUnitaryGroup*, *Stabiliser*, *Stabilizer*, *Subgroup*, *SubgroupLattice*, *SubgroupMembership*, *Supergroup*, *SuzukiGroup*, *SylowSubgroup*, *Symm*, *SymmetricGroup*, *SymplecticGroup*, *ThompsonGroup*, *TitsGroup*, *TransitiveGroup*, *TrivialGroup*, *UpperCentralSeries*]

We enter the cyclic group of order 42.

```
> G:=CyclicGroup(42);
```

$$G := C_{42}$$

We identify it as a small group, the sixth type of order 42.

```
> IdentifySmallGroup(G);
```

$$42, 6$$

We ask if it is Abelian.

```
> IsAbelian(G);
```

true

We ask if it is cyclic.

```
> IsCyclic(G);
```

true

Not very informative. We ask for the elements of the group.

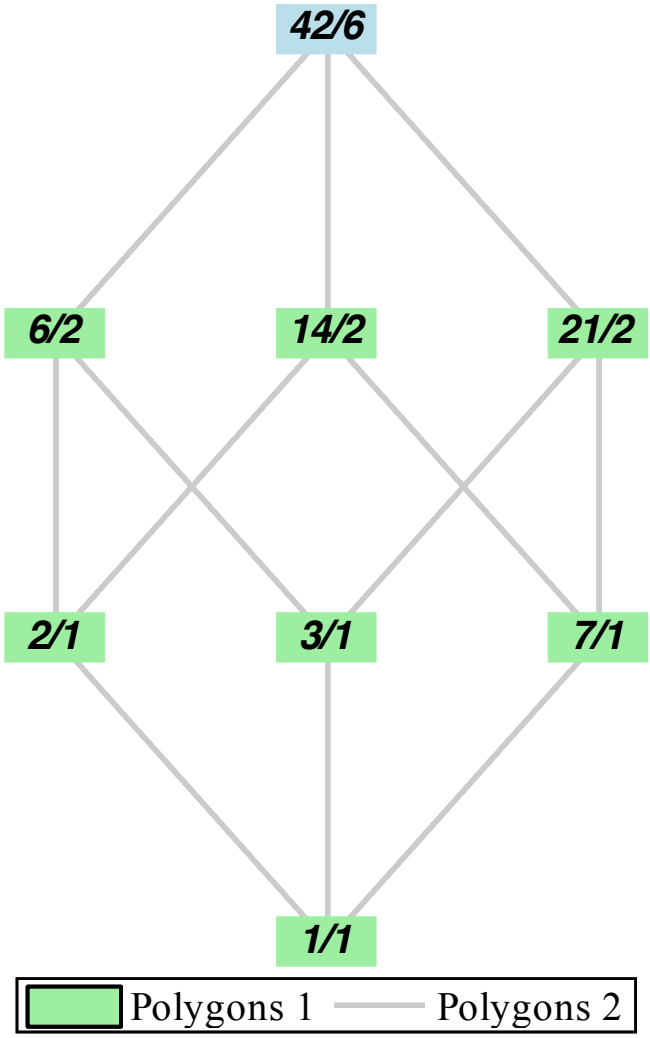
```
> E:=Elements(G);
```

$$E := \{(\,),\ (1, 12, 23, 34, 3, 14, 25, 36, 5, 16, 27, 38, 7, 18, 29, 40, 9, 20, 31, 42, 11, 22, 33, 2, 13, 24, 35, 4, 15, 26, 37, 6, 17, 28, 39, 8, 19, 30, 41, 10, 21, 32),\ (1, 14, 27, 40, 11, 24, 37, 8, 21, 34, 5, 18, 31, 2, 15, 28, 41, 12, 25, 38, 9, 22, 35, 6, 19, 32, 3, 16, 29, 42, 13, 26, 39, 10, 23, 36, 7, 20, 33, 4, 17, 30),\ (1, 18, 35, 10, 27, 2, 19, 36, 11, 28, 3, 20, 37, 12, 29, 4, 21, 38, 13, 30, 5, 22, 39, 14, 31, 6, 23, 40, 15, 32, 7, 24, 41, 16, 33, 8, 25, 42, 17, 34, 9, 26),\ (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42),\ (1, 20, 39, 16, 35, 12, 31, 8, 27, 4, 23, 42, 19, 38, 15, 34, 11, 30, 7, 26, 3, 22, 41, 18, 37, 14, 33, 10, 29, 6, 25, 2, 21, 40, 17, 36, 13, 32, 9, 28, 5, 24),\ (1, 24, 5, 28, 9, 32, 13, 36, 17, 40, 21, 2, 25, 6, 29, 10, 33, 14, 37, 18, 41, 22, 3, 26, 7, 30, 11, 34, 15, 38, 19, 42, 23, 4, 27, 8, 31, 12, 35, 16, 39, 20),\ (1, 26, 9, 34, 17, 42, 25, 8, 33, 16, 41, 24, 7, 32, 15, 40, 23, 6, 31, 14, 39, 22, 5, 30, 13, 38, 21, 4, 29, 12, 37, 20, 3, 28, 11, 36, 19, 2, 27, 10, 35, 18),\ (1, 30, 17, 4, 33, 20, 7, 36, 23, 10, 39, 26, 13, 42, 29, 16, 3, 32, 19, 6, 35, 22, 9, 38, 25, 12, 41, 28, 15, 2, 31, 18, 5, 34, 21, 8, 37, 24, 11, 40, 27, 14),\ (1, 32, 21, 10, 41, 30, 19, 8, 39, 28, 17, 6, 37, 26, 15, 4, 35, 24, 13, 2, 33, 22, 11, 42, 31, 20, 9, 40, 29, 18, 7, 38, 27, 16, 5, 36, 25, 14, 3, 34, 23, 12),\ (1, 38, 33, 28, 23, 18, 13, 8, 3, 40, 35, 30, 25, 20, 15, 10, 5, 42, 37, 32, 27, 22, 17, 12, 7, 2, 39, 34, 29, 24, 19, 14, 9, 4, 41, 36, 31, 26, 21, 16, 11, 6),\ (1, 42, 41, 40, 39, 38, 37, 36, 35, 34, 33, 32, 31, 30, 29, 28, 27, 26, 25, 24, 23, 22, 21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2),\ (1, 6, 11, 16, 21, 26, 31, 36, 41, 4, 9, 14, 19, 24, 29, 34, 39, 2, 7, 12, 17, 22, 27, 32, 37, 42, 5, 10, 15, 20, 25, 30, 35, 40, 3, 8, 13, 18, 23, 28, 33, 38),\ (1, 11, 21, 31, 41, 9, 19, 29, 39, 7, 17, 27, 37, 5, 15, 25, 35, 3, 13, 23, 33)(2, 12, 22, 32, 42, 10, 20, 30, 40, 8, 18, 28, 38, 6, 16, 26, 36, 4, 14, 24, 34),\ (1, 17, 33, 7, 23, 39, 13, 29, 3, 19, 35, 9, 25, 41, 15, 31, 5, 21, 37, 11, 27)(2, 18, 34, 8, 24, 40, 14, 30, 4, 20, 36, 10, 26, 42, 16, 32, 6, 22, 38, 12, 28),\ (1, 21, 41, 19, 39, 17, 37, 15, 35, 13, 33, 11, 31, 9, 29, 7, 27, 5, 25, 3, 23)(2, 22, 42, 20, 40, 18, 38, 16, 36, 14, 34, 12, 32, 10, 30, 8, 28, 6, 26, 4, 24),\ (1, 23, 3, 25, 5, 27, 7, 29, 9, 31, 11, 33, 13, 35, 15, 37, 17, 39, 19, 41, 21)(2, 24, 4, 26, 6, 28, 8, 30, 10, 32, 12, 34, 14, 36, 16, 38, 18, 40, 20, 42, 22),\ (1, 27, 11, 37, 21, 5, 31, 15, 41, 25, 9, 35, 19, 3, 29, 13, 39, 23, 7, 33, 17)(2, 28, 12, 38, 22, 6, 32, 16, 42, 26, 10, 36, 20, 4, 30, 14, 40, 24, 8, 34, 18),\ (1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41)(2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42),\ (1, 33, 23, 13, 3, 35, 25, 15, 5, 37, 27, 17, 7, 39, 29, 19, 9, 41, 31, 21, 11)(2, 34, 24, 14, 4, 36, 26, 16, 6, 38, 28, 18, 8, 40, 30, 20, 10, 42, 32, 22, 12),\ (1, 35, 27, 19, 11, 3, 37, 29, 21, 13, 5, 39, 31, 23, 15, 7, 41, 33, 25, 17, 9)(2, 36, 28, 20, 12, 4, 38, 30, 22, 14, 6, 40, 32, 24, 16, 8, 42, 34, 26, 18, 10),\ (1, 39, 35, 31, 27, 23, 19, 15, 11, 7, 3, 41, 37, 33, 29, 25, 21, 17, 13, 9, 5)(2, 40, 36, 32, 28, 24, 20, 16, 12, 8, 4, 42, 38, 34, 30, 26, 22,$$

18, 14, 10, 6), (1, 41, 39, 37, 35, 33, 31, 29, 27, 25, 23, 21, 19, 17, 15, 13, 11, 9, 7, 5, 3)(2, 42, 40, 38, 36, 34, 32, 30, 28, 26, 24, 22, 20, 18, 16, 14, 12, 10, 8, 6, 4), (1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 3, 7, 11, 15, 19, 23, 27, 31, 35, 39)(2, 6, 10, 14, 18, 22, 26, 30, 34, 38, 42, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40), (1, 9, 17, 25, 33, 41, 7, 15, 23, 31, 39, 5, 13, 21, 29, 37, 3, 11, 19, 27, 35)(2, 10, 18, 26, 34, 42, 8, 16, 24, 32, 40, 6, 14, 22, 30, 38, 4, 12, 20, 28, 36), (1, 10, 19, 28, 37, 4, 13, 22, 31, 40, 7, 16, 25, 34)(2, 11, 20, 29, 38, 5, 14, 23, 32, 41, 8, 17, 26, 35)(3, 12, 21, 30, 39, 6, 15, 24, 33, 42, 9, 18, 27, 36), (1, 16, 31, 4, 19, 34, 7, 22, 37, 10, 25, 40, 13, 28)(2, 17, 32, 5, 20, 35, 8, 23, 38, 11, 26, 41, 14, 29)(3, 18, 33, 6, 21, 36, 9, 24, 39, 12, 27, 42, 15, 30), (1, 28, 13, 40, 25, 10, 37, 22, 7, 34, 19, 4, 31, 16)(2, 29, 14, 41, 26, 11, 38, 23, 8, 35, 20, 5, 32, 17)(3, 30, 15, 42, 27, 12, 39, 24, 9, 36, 21, 6, 33, 18), (1, 34, 25, 16, 7, 40, 31, 22, 13, 4, 37, 28, 19, 10)(2, 35, 26, 17, 8, 41, 32, 23, 14, 5, 38, 29, 20, 11)(3, 36, 27, 18, 9, 42, 33, 24, 15, 6, 39, 30, 21, 12), (1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40)(2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 41)(3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42), (1, 40, 37, 34, 31, 28, 25, 22, 19, 16, 13, 10, 7, 4)(2, 41, 38, 35, 32, 29, 26, 23, 20, 17, 14, 11, 8, 5)(3, 42, 39, 36, 33, 30, 27, 24, 21, 18, 15, 12, 9, 6), (1, 13, 25, 37, 7, 19, 31)(2, 14, 26, 38, 8, 20, 32)(3, 15, 27, 39, 9, 21, 33)(4, 16, 28, 40, 10, 22, 34)(5, 17, 29, 41, 11, 23, 35)(6, 18, 30, 42, 12, 24, 36), (1, 19, 37, 13, 31, 7, 25)(2, 20, 38, 14, 32, 8, 26)(3, 21, 39, 15, 33, 9, 27)(4, 22, 40, 16, 34, 10, 28)(5, 23, 41, 17, 35, 11, 29)(6, 24, 42, 18, 36, 12, 30), (1, 25, 7, 31, 13, 37, 19)(2, 26, 8, 32, 14, 38, 20)(3, 27, 9, 33, 15, 39, 21)(4, 28, 10, 34, 16, 40, 22)(5, 29, 11, 35, 17, 41, 23)(6, 30, 12, 36, 18, 42, 24), (1, 31, 19, 7, 37, 25, 13)(2, 32, 20, 8, 38, 26, 14)(3, 33, 21, 9, 39, 27, 15)(4, 34, 22, 10, 40, 28, 16)(5, 35, 23, 11, 41, 29, 17)(6, 36, 24, 12, 42, 30, 18), (1, 37, 31, 25, 19, 13, 7)(2, 38, 32, 26, 20, 14, 8)(3, 39, 33, 27, 21, 15, 9)(4, 40, 34, 28, 22, 16, 10)(5, 41, 35, 29, 23, 17, 11)(6, 42, 36, 30, 24, 18, 12), (1, 7, 13, 19, 25, 31, 37)(2, 8, 14, 20, 26, 32, 38)(3, 9, 15, 21, 27, 33, 39)(4, 10, 16, 22, 28, 34, 40)(5, 11, 17, 23, 29, 35, 41)(6, 12, 18, 24, 30, 36, 42), (1, 36, 29, 22, 15, 8)(2, 37, 30, 23, 16, 9)(3, 38, 31, 24, 17, 10)(4, 39, 32, 25, 18, 11)(5, 40, 33, 26, 19, 12)(6, 41, 34, 27, 20, 13)(7, 42, 35, 28, 21, 14), (1, 8, 15, 22, 29, 36)(2, 9, 16, 23, 30, 37)(3, 10, 17, 24, 31, 38)(4, 11, 18, 25, 32, 39)(5, 12, 19, 26, 33, 40)(6, 13, 20, 27, 34, 41)(7, 14, 21, 28, 35, 42), (1, 15, 29)(2, 16, 30)(3, 17, 31)(4, 18, 32)(5, 19, 33)(6, 20, 34)(7, 21, 35)(8, 22, 36)(9, 23, 37)(10, 24, 38)(11, 25, 39)(12, 26, 40)(13, 27, 41)(14, 28, 42), (1, 29, 15)(2, 30, 16)(3, 31, 17)(4, 32, 18)(5, 33, 19)(6, 34, 20)(7, 35, 21)(8, 36, 22)(9, 37, 23)(10, 38, 24)(11, 39, 25)(12, 40, 26)(13, 41, 27)(14, 42, 28), (1, 22)(2, 23)(3, 24)(4, 25)(5, 26)(6, 27)(7, 28)(8, 29)(9, 30)(10, 31)(11, 32)(12, 33)(13, 34)(14, 35)(15, 36)(16, 37)(17, 38)(18, 39)(19, 40)(20, 41)(21, 42)}

OK, these are permutations, the topic of our next chapter, so understanding will come later. We draw the subgroup lattice.

```
> DrawSubgroupLattice(G,'labels = ids');
```



The first number is the order of the group, the second is which group it is in the table for that order.

```
> SmallGroup(42,6);
```

⟨(1, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 41, 39, 37, 35, 33, 31, 29, 27, 25, 23, 21, 19, 17, 15, 13, 11, 9, 7, 5, 3)⟩

This one permutation is the generator of the group. Again, it is a permutation.

```
> SmallGroup(3,1);
```

⟨(1, 2, 3)⟩

The same here.