LINEAR AND ANGULAR MOMENTUM, PRINCIPLE OF IMPULSE AND MOMENTUM

Today’s Objectives:
Students will be able to:
1. Develop formulations for the linear and angular momentum of a body.
2. Apply the principle of linear and angular impulse and momentum.

In-Class Activities:
• Check Homework
• Reading Quiz
• Applications
• Linear and Angular Momentum
• Principle of Impulse and Momentum
• Concept Quiz
• Group Problem Solving
• Attention Quiz
READING QUIZ

1. The angular momentum of a rotating two-dimensional rigid body about its center of mass \( G \) is ____________.
   
   A) \( m v_G \)  
   C) \( m \omega \)  
   B) \( I_G v_G \)  
   D) \( I_G \omega \)

2. If a rigid body rotates about a fixed axis passing through its center of mass, the body’s linear momentum is ____________.
   
   A) a constant  
   C) \( m v_G \)  
   B) zero  
   D) \( I_G \omega \)
The swing bridge opens and closes by turning using a motor located under the center of the deck at A that applies a torque $M$ to the bridge.

If the bridge was supported at its end B, would the same torque open the bridge in the same time, or would it open slower or faster?

What are the benefits of making the bridge with the variable depth (thickness) substructure as shown?
As the pendulum of the Charpy tester swings downward, its angular momentum and linear momentum both increase. By calculating its momenta in the vertical position, we can calculate the impulse the pendulum exerts when it hits the test specimen.

As the pendulum rotates about point O, what is its angular momentum about point O?
The space shuttle has several engines that exert thrust on the shuttle when they are fired. By firing different engines, the pilot can control the motion and direction of the shuttle.

If only engine A is fired, about which axis does the shuttle tend to rotate?
LINEAR AND ANGULAR MOMENTUM
(Section 19.1)

The **linear momentum** of a rigid body is defined as

\[ L = m \, v_G \]

This equation states that the linear momentum vector \( L \) has a magnitude equal to \((mv_G)\) and a direction defined by \( v_G \).

The **angular momentum** of a rigid body is defined as

\[ H_G = I_G \, \omega \]

Remember that the direction of \( H_G \) is perpendicular to the plane of rotation.
Translation.

When a rigid body undergoes rectilinear or curvilinear translation, its angular momentum is zero because $\omega = 0$.

Therefore,

\[ L = m v_G \]

and

\[ H_G = 0 \]
Rotation about a fixed axis.

When a rigid body is rotating about a fixed axis passing through point O, the body’s linear momentum and angular momentum about G are:

\[ L = m v_G \]

\[ H_G = I_G \omega \]

It is sometimes convenient to compute the angular momentum of the body about the center of rotation O.

\[ H_O = (r_G \times m v_G) + I_G \omega = I_O \omega \]
General plane motion.

When a rigid body is subjected to general plane motion, both the linear momentum and the angular momentum computed about G are required.

\[ L = m v_G \]
\[ H_G = I_G \omega \]

The angular momentum about point A is

\[ H_A = I_G \omega + m v_G (d) \]
As in the case of particle motion, the principle of impulse and momentum for a rigid body is developed by combining the equation of motion with kinematics. The resulting equations allow a **direct solution to problems involving force, velocity, and time**.

**Linear impulse-linear momentum equation:**

\[
L_1 + \sum_{t_1}^{t_2} \int F \ dt = L_2 \quad \text{or} \quad (m \mathbf{v}_G)_1 + \sum_{t_1}^{t_2} \int F \ dt = (m \mathbf{v}_G)_2
\]

**Angular impulse-angular momentum equation:**

\[
(H_G)_1 + \sum_{t_1}^{t_2} \int M_G \ dt = (H_G)_2 \quad \text{or} \quad I_G \omega_1 + \sum_{t_1}^{t_2} \int M_G \ dt = I_G \omega_2
\]
PRINCIPLE OF IMPULSE AND MOMENTUM (continued)
The previous relations can be represented graphically by drawing the impulse-momentum diagram.

To summarize, if motion is occurring in the x-y plane, the linear impulse-linear momentum relation can be applied to the x and y directions and the angular momentum-angular impulse relation is applied about a z-axis passing through any point (i.e., G). Therefore, the principle yields three scalar equations describing the planar motion of the body.
PROCEDURE FOR ANALYSIS

• **Establish** the x, y, z inertial frame of reference.

• Draw the impulse-momentum **diagrams** for the body.

• Compute $I_G$, as necessary.

• Apply the **equations of impulse and momentum** (one vector and one scalar or the three scalar equations).

• If more than three unknowns are involved, **kinematic** equations relating the velocity of the mass center $G$ and the angular velocity $\omega$ should be used to furnish additional equations.
EXAMPLE

**Given:** The 300 kg wheel has a radius of gyration about its mass center O of $k_O = 0.4$ m. The wheel is subjected to a couple moment of 300 N\(\cdot\)m.

**Find:** The angular velocity after 6 seconds if it starts from rest and no slipping occurs.

**Plan:** Time as a parameter should make you think Impulse and Momentum! Since the body rolls without slipping, point A is the center of rotation. Therefore, applying the angular impulse and momentum relationships along with kinematics should solve the problem.
Solution:

Impulse-momentum diagram:

Kinematics: \((v_G)_2 = r \cdot \omega_2\)

Impulse & Momentum: \((H_A)_1 + \sum \int_{t_1}^{t_2} M_A \, dt = (H_A)_2\)

\[0 + M \cdot t = m(v_G)_2 \cdot r + I_G \cdot \omega_2 = m \cdot r^2 \cdot \omega_2 + m(k_O)^2 \cdot \omega_2 = m \cdot \{r^2 + (k_O)^2\} \cdot \omega_2\]

\[\omega_2 = \frac{M \cdot t}{m \cdot \{r^2 + (k_O)^2\}} = \frac{300 \cdot (6)}{300 \cdot (0.6^2 + 0.4^2)} = 11.5 \text{ rad/s}\]
CONCEPT QUIZ

1. If a slab is rotating about its center of mass $G$, its angular momentum about any arbitrary point $P$ is __________ its angular momentum computed about $G$ (i.e., $I_G \omega$).

   A) larger than        B) less than
   C) the same as       D) None of the above

2. The linear momentum of the slab in question 1 is __________.

   A) constant        B) zero
   C) increasing linearly with time
   D) decreasing linearly with time
GROUP PROBLEM SOLVING

Find: The angular velocity of gear B after 5 seconds if the gears start turning from rest.

Plan: Time is a parameter, thus Impulse and Momentum is recommended. First, relate the angular velocities of the two gears using kinematics. Then apply angular impulse and momentum to both gears.

Given: A gear set with:
- \( m_A = 10 \text{ kg} \)
- \( m_B = 50 \text{ lb} \)
- \( k_A = 0.08 \text{ m} \)
- \( k_B = 0.15 \text{ m} \)
- \( M = 10 \text{ N} \cdot \text{m} \)
GROUP PROBLEM SOLVING (continued)

Solution:

Impulse-momentum diagrams: Note that the initial momentum is zero for both gears.

Gear A:

\[ \begin{align*}
W_A t &= M t + A_x t + A_y t + F_t \\
I_A \omega_A &= r_A
\end{align*} \]

Gear B:

\[ \begin{align*}
W_B t &= F t + B_x t + B_y t \\
I_B \omega_B &= r_B
\end{align*} \]
Kinematics: \( r_A \omega_A = r_B \omega_B \)

Angular impulse & momentum relation:

For gear A: \( M \ t - (F \ t) \ r_A = I_A \ \omega_A \)

For gear B: \( (F \ t) \ r_B = I_B \ \omega_B \) \( \Rightarrow \) \( (F \ t) = (I_B \ \omega_B) / r_B \)

Combining the two equations yields:

\[ M \ t = I_A \ \omega_A + (r_A/r_B) \ I_B \omega_B \]

Substituting from kinematics for \( \omega_A = (r_B/r_A) \omega_B \), yields

\[ M \ t = \omega_B \left[ (r_B/r_A) \ I_A + (r_A/r_B) \ I_B \right] \quad \text{eqn (1)} \]
Therefore, \( \omega_B = 72.4 \text{ rad/s} \)

and \( \omega_A = \left( \frac{r_B}{r_A} \right) \omega_B = \left( \frac{0.2}{0.1} \right) 72.4 = 144 \text{ rad/s} \)
ATTENTION QUIZ

1. If a slender bar rotates about end A, its angular momentum with respect to A is?
   A) \( \frac{1}{12} m l^2 \omega \)   B) \( \frac{1}{6} m l^2 \omega \)
   C) \( \frac{1}{3} m l^2 \omega \)   D) \( m l^2 \omega \)

2. As in the principle of work and energy, if a force does no work, it does not need to be shown on the impulse and momentum diagram/equation.
   A) False   B) True
   C) Depends on the case   D) No clue!
End of the Lecture

Let Learning Continue