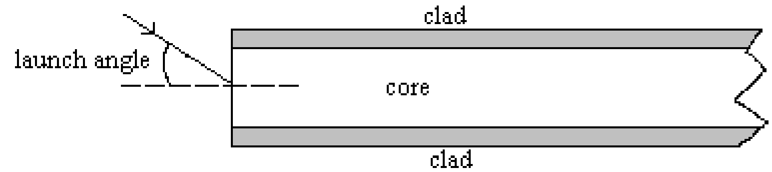
PHYSICS 252: SUPPLEMENTAL PROBLEMS

PART 1

1A. (Optical Fiber)
A typical optical fiber has a cylindrical core with an index of refraction of 1.62 wrapped by a cladding jacket whose index of refraction is 1.52. The diameter of the core is 50 μm (microns). Light traveling in air is launched into one end of the fiber at a launch angle as defined in the sketch. The light undergoes total internal reflection at the core-clad interfaces and "zig-zags" down the length of the fiber.

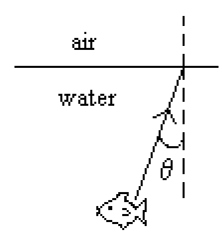


(a) Find the critical angle for the light once it is in the core so that total internal reflection (TIR) occurs at the core-clad interfaces.
(b) Find the launch angle that will yield this critical angle.
(c) Will TIR occur at the core-clad interfaces for launch angles greater than or smaller than the launch angle found in (b)? (Be careful. Pick an angle and see or explain your reasoning.)

For parts (d) through (g), consider light entering the fiber at a launch angle of 20°.
(d) What are the angles of incidence and reflection at the core-clad interfaces?
(e) Find the horizontal distance traveled by a ray that reflects from the bottom of the core, goes to the top of the core, reflects and comes back down to the bottom of the core. Note that this ray has made 2 reflections or "bounces".
(f) If the fiber is 1 km long, approximately how many bounces does a light ray make in traveling the length of the fiber?
(g) Approximately how long does it take for the light to travel this 1 km? (Use the core index to figure the speed of the light.)

ANSWERS: (a) 69.76° (b) 34.09° (c) Tell me. (d) 77.81° (e) 0.463 mm (f) ~4.3 million!! (g) ~5.5 μs

1B. (Apparent Depth and Paraxial/Nonparaxial Rays)
A fish is resting one meter below the surface in a pool of water (n = 1.33). Find the depth of the fish's image below the surface if the image is viewed with rays that strike the water-air interface at an angle of $\theta = 5°$. Repeat for angles of 10°, 15°, etc. up to the critical angle. Also, find the depth for normal incidence ($\theta = 0$)



ANSWERS: The critical angle is 48.753°. You should find the following image depths:

θ_1 (°)	0	5	10	15	20	25	30	35	40	45	48.75
depth (cm)	100	75.0	74.3	73.1	71.3	68.6	64.8	59.4	50.9	36.1	1.2

Note that for rays that are paraxial (i.e. the small angle approximation $\sin\theta \approx \tan\theta \approx \theta$ is valid), the image appears at about the same depth of 70 to 75 cm below the surface. For nonparaxial rays, there is a much more dramatic change in apparent depth as the incident angle changes by $5°$. For an angle near the critical angle, the image appears right below the surface. However, it will be blurry since your eyes will also be receiving rays at other close by angles which have significantly different image depths. You may also be wondering about the drastic change in image depth from normal incidence to $5°$. While it is true that the ray normal to the surface will make you see the object at its actual depth of 100 cm, your eye will also be receiving rays at other small, nonzero angles. All of these rays put the image at about 75 cm below the surface. That is, as soon as the angle of incidence is a bit bigger than zero, the image depth jumps to around 75 cm. Try it and see. If you pick an angle of 0.1 °, for instance, you should find an image depth of 75.2 cm.

1C. (Designing Lenses I)
Design lenses out of glass with an index of refraction of 1.5 with the following characteristics. To design a lens, you must specify the radii of curvature of the two surfaces.

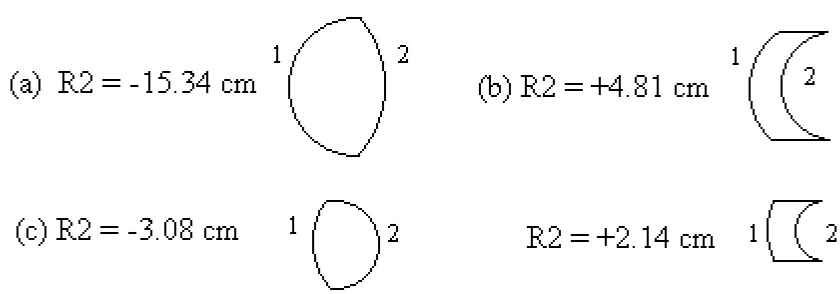
(a) a positive lens with f = +10 cm, shaped like a "football"
(b) a positive lens with f = +10 cm, shaped like a "C"
(c) a plano-convex lens with f = +10 cm
(d) a negative lens with f = -15 cm, shaped like an "hour glass"
(e) a negative lens with f = -15 cm, shaped like a "C"
(f) a plano-concave lens with f = -15 cm

ANSWERS: (a) Many designs: R_1 is +, R_2 is - (b) Many designs: R_1& R_2 are + with $R_2 > R_1$ (c) R_1= +5 cm, R_2= ∞ or R_1= ∞, R_2= -5 cm (d) Many designs: R_1 is -, R_2 is + (e) Many designs: R_1& R_2 are + with $R_1 > R_2$ (f) R_1= -7.5 cm, R_2= ∞ or R_1= ∞, R_2= +7.5 cm

1D. (Designing Lenses II)
A lens with an index of refraction of 1.61 has a front surface with an radius of curvature of 14 cm.

(a) Find the radius of curvature of the second surface if the lens is to act as a positive lens with a focal length of 12 cm. Sketch the lens.
(b) Find the radius of curvature of the second surface if the lens is to act as a negative lens with a focal length of -12 cm. Sketch the lens.
(c) Repeat (a) & (b) if the lenses are to be used in water (n=1.333).

ANSWERS:



1E. (Thin Lenses & Fish Eyes)
Let's say you go fishing and land a monster bass. It is 35 cm long. You take a picture of the bass using your homemade camera that consists of a single plano-convex lens with a refractive index of 1.55 and a piece of film. The image of the fish is reduced to 35 mm to fit on the film when the bass is 80 cm from the lens.

(a) What magnification is the lens providing?
(b) How far should the film be from the lens?
(c) What is the focal length of the lens?
(d) Show a ray trace of the system. Show the object, lens, image, and indicate where you should place the film. Your ray trace need not be to scale but it should show whether the image is magnified or demagnified.
(e) Using the lens maker's equation, find the radius of curvature of the curved side of the lens. Make sure that the sign of the radius corresponds to the lens shown in (d).
(f) Suppose that you make another plano-convex lens out of the same glass so that its focal length underwater is the same as the focal length of the first lens used in air. What radius of curvature must the curved side have? (The refractive index of water is 1.333)
(g) Which lens is more rounded: the first lens used in the air or the second lens used underwater?
(h) While looking at your bass, you notice that its eyes bulge out and are very rounded. What do you think might be one reason for this?

ANSWERS: (a) -0.1 (b) 80 mm (c) 72.7 mm (d) Show me. (e) ± 40 mm (f) ± 11.8 mm (g)&(h) Tell me.

1F. (Compound Microscope)
Design a compound microscope with a total magnifying power of 500x. The distance between the objective and eyepiece lenses must be between 12 cm and 16 cm.
Tell me your design by specifying the following quantities:

total magnification	distance between lenses
objective object distance	eyepiece object distance
objective image distance	eyepiece image distance
objective focal length	eyepiece focal length
objective magnification	eyepiece magnification

ANSWER: There are many possible designs. Demonstrate to me that your design is consistent with the equations that pertain to the compound microscope. Be careful of signs!! (Typical eyepiece magnifications are between 5x and 10x. You also may want to choose an eyepiece image distance of -25 cm for optimum eyepiece magnification and ease in viewing.)

PART 2

2A. (Diffraction Grating)
The grating that you used in class to look at the white light spectrum of the tungsten lamp has a grating constant of 530 lines/mm. Assume that collimated white light hits the grating with the grating oriented perpendicular to the light's direction of travel. Also assume that the visible spectrum extends from 390 nm to 780 nm.

(a) What is the slit spacing, *d*, for the grating?
(b) Find the total angle that the visible spectrum subtends in the first order diffraction. Do this by finding the angle of the first order 390 nm light, then finding the angle for the first order 780 nm light, and then subtracting the two angles.
(c) Find the total angle that the visible spectrum subtends in the second order diffraction.
(d) Find the angle for the third order 390 nm violet light.
(e) If you look closely at the angle in (d), you should see that this violet light lies somewhere in the second order spectrum. Find the wavelength in the second order spectrum that has the same angle as the third order 390 nm light. What color is this second order light?

{Note: It turns out that the only "pure" visible spectrum for this grating is the first order spectrum. All higher order spectrums have some wavelengths from even higher order diffractions mixed in with their wavelengths. The higher order colors are not as intense so that the lower order wavelengths dominate. However, this mixing can be a problem in sensitive spectroscopic work. For this reason, the profiles of the groove edges on reflection gratings are often polished so as to suppress higher order diffraction intensities even more. The production of this profile is called "blazing". In addition to reducing the intensities of higher order diffractions, a blazed grating also takes much of the light from the zeroth order diffraction and puts into one of the lower order patterns (usually the first order). Recall that the zeroth order line is useless for spectroscopic work since all of the wavelengths are at the same angle of $\theta=0$. Thus, a blazed grating diffracts the majority of the light into one order and the spectroscopic work is performed over the angles of this order.}

ANSWERS: (a) 1.887 μm (b) $\Delta\theta$ = 12.49° (c) $\Delta\theta$ = 31.34° (d) 38.32° (e) 585 nm, tell me color.

2B. (Antireflection Coatings)
Magnesium fluoride (n = 1.38) is often used as an antireflecting coating on lenses. Find the minimum thickness necessary to use on a glass lens with n = 1.5 so that the loss of light due to reflection is minimized. Since white light is comprised of a spread of wavelengths, design the coating for the midpoint of the range (λ = 580 nm).

{Note: The coating will work best for wavelengths near 580 nm. Thus, the coating will reflect more red and violet light than yellow and green light. For this reason, MgF_2 coatings appear purple. Also, since energy must be conserved, the light that is not reflected must be transmitted through the lens. So an antireflection coating really puts more light through the lens compared to the uncoated lens. Perhaps a better term for the coating would be a "transmission enhancement" coating.}

ANSWER: minimum thickness = 105 nm

2C. (Diffraction Limited Resolution)
The Moon has a diameter of 3476 km and is approximately 384000 km away from the Earth. We see the Moon due to reflected sunlight. Assume that the light is at a wavelength of 500 nm, the peak wavelength from the Sun.

(a) What angle does the Moon subtend in the sky? (Express your answer in minutes of arc.)
(b) You are viewing the Moon with your naked eye and your pupil is dilated to 4 mm. What is the smallest angle that you can resolve? (Express your answer in seconds of arc.)
(c) What is the smallest separation distance on the Moon that you can resolve? Can you resolve two craters on the Moon that are 2 km apart?

You now view the Moon through the Hale reflector telescope at Mount Palomar. The telescope has a diameter of 5 m. Assume that the telescope's performance is diffraction limited.
(d) What is the smallest angle that the telescope can resolve? (Express your answer in seconds of arc.)
(e) Assume that the magnification of the telescope is 1000x. What angle does this become for your eye?
(f) Assuming that your pupil is still 4 mm wide when you look through the telescope, will you be able to fully utilize the telescope's resolving capability? Or will your eye limit the resolution of the image? Explain.
(g) Can you now resolve the two craters? What is the smallest separation distance on the Moon that you can resolve now?

{Note: The resolution of the Hale telescope is limited not by diffraction but by atmospheric blurring. The usual resolution angle is between 0.1 and 1 sec of arc. While the Hubble space telescope's mirror has a smaller diameter of 2.4 m, it truly is diffraction limited and has a resolution angle of approximately 0.05 sec of arc (now that they have corrected the original problem with the mirror).}

ANSWERS: (a) 31.12 arcmin (b) 23.65 arcsec (c) 44 km (yes or no?) (d) 0.025 arcsec (e) 25 arcsec (f) Hint: Compare the angles in (b) and (e). (g) yes or no?, 46.85 m

PART 3

3A. (Blackbody Radiation & You)
You are sitting in your room doing physics homework. Your skin temperature is 95°F (a typical skin temperature). Let's say that your skin surface area is 12000 cm^2 and that you radiate as a perfect blackbody.

(a) At what wavelength does your thermal radiation peak?
(b) In what portion of the electromagnetic spectrum is this radiation?
(c) What is the energy of a photon at this peak wavelength? Express the energy in eV and Joules.
(d) Assume that you radiate approximately 10 W at this peak wavelength. What is the photon rate of this radiation?
(e) How much total power do you radiate?
(f) Assume that the walls of your room are at 72°F. How much power are you absorbing from the walls?
(g) Do you absorb or emit more power? How much more?

{Ponder this. If you are emitting more power, from where are you getting the extra energy? Welcome to the world of biophysics!}

ANSWERS: (a) 9.41 μm (b) Tell me. (c) 0.132 eV = 2.1×10^{-20} J (d) 4.76×10^{20} photons/sec (e) 614 W (f) 518 W (g) Tell me.

3B. (Photomultiplier Tube)
A PMT with a GaAs photocathode is used to measure the light intensity in an experiment. The work function of GaAs is 1.424 eV. The PMT has 9 dynodes. You are interested in measuring visible wavelengths.

(a) What is the cutoff wavelength for this PMT?
(b) Will this PMT work in the visible portion of the spectrum? Why?

Assume you are measuring 550 nm light at one point in the experiment and that 20 picoWatts of the light is incident upon the detector. The photocathode has a quantum efficiency of 22% at this wavelength.
(c) What is the maximum kinetic energy of one of the photoelectrons?
(d) What would be the stopping potential required to stop any photoelectrons from being detected?
(e) Find the number of photoelectrons leaving the cathode in 1 second.
(f) The secondary emission ratio of a dynode is 4, i.e. each dynode emits an average of 4 electrons for each incident electron. Find the total number of electrons collected at the anode each second for this 550 nm light.
(g) What size current is produced by these electrons?

ANSWERS: (a) 873 nm (b) Tell me. (c) 0.836 eV (d) 0.836 V (e) $1.22\text{x}10^7$ e^-/s (f) $3.20\text{x}10^{12}$ e^-/s (g) 0.51 μA

3C. (Bohr Model of Hydrogen)
Consider a Hydrogen atom in the ground state. Using Bohr's model, answer the following questions.

(a) What is the minimum amount of energy necessary to ionize the atom (i.e. to remove the electron from the atom)?
(b) What wavelength must a photon have to do this? In what portion of the spectrum is this photon?
(c) Suppose instead that the atom absorbs a photon with a wavelength of 102.8 nm. To what state is the atom excited?
(d) What is the diameter of the electron's orbit in this state? How much volume (expressed in cubic centimeters) does the atom occupy?
(e) What are the possible decay schemes of the atom back to the ground state? For each scheme, list the energies and wavelengths of the emitted photons and state in what portion of the spectrum each photon belongs.
(f) Can the ground state atom absorb visible light? Why or why not?

ANSWERS: (a) Tell me. (b) 91.4 nm, tell me (c) n=3 (2nd excited state) (d) Diameter=9.54 Å, You figure out the volume. You can assume the atom is a sphere. (e) Scheme 1: $n=3 \rightarrow n=1$ (λ=102.8 nm) Tell me energy & portion of spectrum. Scheme 2: $n=3 \rightarrow n=2$ (λ=657.7 nm) then $n=2 \rightarrow n=1$ (λ=121.8 nm) Tell me energies & portions of spectrum. (f) Tell me. (Hint: Find the smallest amount of energy that the ground state atom can absorb.)

3D. (X-Ray Production)
Suppose you wish to produce 0.07-nm x-rays in the lab.
(a) What is the minimum voltage you must use in accelerating the electrons?
(b) Does this minimum voltage depend on the target material?
(c) Suppose that this radiation is to correspond to a K_α line so as to maximize the intensity at this wavelength. Does choice of target material matter now?

ANSWERS: (a) 17.7 kV (b) no (c) yes

PART 4

4A. (Special Relativity Meets The Original Star Trek)
Captain Kirk leaves the Enterprise in a shuttlecraft and speeds away at 0.7*c* relative to the Enterprise. Mr. Spock, back on board the Enterprise, monitors the shuttlecraft with the ship's sensors.
(a) Kirk measures the length of the shuttlecraft to be 10 m. What length does Mr. Spock measure? Who measures the *proper* length of the shuttlecraft?
(b) A Romulan has sneaked on board the shuttlecraft. Kirk sees him and stuns him with a phaser blast. Kirk measures the blast to last 0.5 seconds. How long does the blast last as seen by Spock? Who measures the *proper* time interval of the phaser blast?
(c) A Romulan ship suddenly decloaks and approaches the shuttlecraft at 0.8*c* as measured by Kirk from a direction directly opposite that of the Enterprise. What is the speed of the Romulan ship as measured by Spock? Is the ship approaching or receding from the Enterprise? Explain your reasoning. [Draw a *SS'* diagram of the situation. Clearly label which ship is frame *S*, which is *S'*, and which ship is the object that is moving relative to frames *S* & *S'*. Make sure that the directions of the ships and the signs of the velocities are consistent. Use the appropriate velocity transform equation.]
(d) Kirk, being the hero that he is, zooms towards the Romulans. The rest mass of the shuttle with Kirk on board and no fuel is 2500 kg. Kirk has 250 kg of fuel in the form of equal amounts of matter and antimatter. He combines all of the fuel at once. Assuming that the energy transfer method from fuel to shuttle kinetic energy is 100% efficient, what is the shuttle's new speed as measured by Spock? (Hint: Find the initial kinetic energy of the shuttle. Add the energy gained from the fuel to get the final kinetic energy.)
(e) Kirk fires the shuttle's phaser (a stream of photons) at the Romulan ship. How fast does Kirk see the phaser energy travel towards the Romulans? How fast do the Romulans see the phaser energy traveling toward them? How fast does Spock see the phaser energy traveling toward the Romulans?

ANSWERS: (a) 7.14 m, Tell me. (b) 0.7 s, Tell me. (c) 0.227*c*, Tell me if Romulans are approaching or receding as seen by Spock and explain your reasoning. (d) 0.760*c* (e) Tell me, tell me, tell me.

4B. (Relativistic Energy & Momentum)
An electron is accelerated from rest to a speed where its total energy is equal to twice its rest energy.
(a) How much energy did it take to do this?
(b) How fast is the electron traveling?
(c) Find the momentum of the electron in units of MeV/*c* and kg-m/s.

ANSWERS: (a) 0.511 MeV (b) 0.866*c* (c) 0.885 MeV/*c* = $4.7\text{x}10^{-22}$ kg-m/s

4C. (Time Dilation & Length Contraction)
After being accelerated to 0.866*c*, the electron in the above problem leaves an exit port and slams into a metal target. The distance from the exit port to the target is 10 m as measured by a scientist in the lab.
(a) How long does it take for the e^- to travel from the exit port to the target as measured by the scientist?
(b) How long does it take as measured by the e^-? How far is it from the exit port to the target as seen by the e^-?
(c) Is the distance from the exit port to the target measured by the scientist a proper or nonproper length? Is the time measured by the scientist for the e^- to travel this distance a proper or nonproper time interval?
(d) Is the distance from the exit port to the target measured by the e^- a proper or nonproper length? Is the time measured by the e^- for it to travel this distance a proper or nonproper time interval?

ANSWERS: (a) 38.5 ns (b) 19.25 ns, 5 m (c) proper length, nonproper time interval (d) nonproper length, proper time interval

PART 5

5A. (Isotopes of Hydrogen)
The three isotopes of hydrogen are commonly called hydrogen (^{1}H), deuterium (^{2}H), and tritium (^{3}H). Hydrogen and deuterium are stable while tritium decays via β^- emission with a half-life of 12.33 years.

(a) Find the binding energies for the 3 isotopes.
(b) Find the binding energies *per nucleon* for the 3 isotopes.
(c) What does tritium become when it decays? Write down the decay equation.
(d) Is the tritium daughter nucleus stable or unstable?
(e) What is the maximum kinetic energy (in eV and Joules) that the emitted electron could have from the radioactive decay of tritium?
(f) Suppose a sample initially has 10^{18} tritium atoms. What is the initial activity of the sample (in dec/sec and in Curie)?
(g) How long would it take for the activity to reduce to one tenth of its initial level?

ANSWERS: (a) 0, 2.224 MeV, 8.482 MeV (b) 0, 1.112 MeV, 2.827 MeV (c) Tell me. (d) Tell me. (Look at Table A.3 in book Appendix A.) (e) 18.63 keV = $2.98\text{x}10^{-15}$ J (f) $1.782\text{x}10^{9}$ dec/s = 48 milliCi (g) 40.97 yrs

5B. (Fission as an Energy Source)
Let's find out how much energy we can get out of the fission of 1 gram of ^{235}U.
(a) Find the number of nuclei in 1 gram of U-235.
(b) When each nucleus splits, 208 MeV of energy is released. Find how much energy is released when the 1 gram of material undergoes fission.
(c) Convert this energy to Joules.
(d) Convert this energy to kW·hr.
(e) Let's say your house uses an average of 1000 W of power, i.e. the average rate that your house consumes energy is 1000 J/s. How many years could you live in your house using the energy from this 1 gram?
(f) Find how many barrels of oil you would need to burn to produce this same amount of energy. (1 barrel of oil yields 6.3 billion Joules)
(g) Find how much coal you would have to burn to produce this same amount of energy. (1 lb. of coal produces 14.5 million Joules)

ANSWER: (a) $2.56\text{x}10^{21}$ nuclei (b) $5.32\text{x}10^{23}$ MeV (c) $8.5\text{x}10^{10}$ J (d) 23611 kW·hr (e) 2.7 yrs (f) 13.5 barrels (g) 2.9 tons