Part 5: Nuclear Physics

5.1. The Nucleus

Z = atomic number = number of protons

N = neutron number = number of neutrons

A = mass number = $Z + N$

Representations:

$^{A}_{Z}X$ or X-A where X is chemical symbol of element

Examples: $^{12}_{6}\mathrm{C}$ or C-12 , $^{239}_{93}\mathrm{Pu}$ or Pu-239

Isotopes:
same Z, different N and A

Examples: $^{11}_{6}\mathrm{C}$ $^{12}_{6}\mathrm{C}$ $^{13}_{6}\mathrm{C}$ $^{14}_{6}\mathrm{C}$

Charge of Nucleus

$Q = Ze = Z\,(1.6\text{x}10^{-19}\ \mathrm{C})$

Mass of Nucleus

common unit: [unified mass unit = u] also known as [atomic mass unit = amu] or [Dalton = Da]

definition: mass of neutral, ground state C-12 atom = 12 u

conversions: [1 u = $1.660559\text{x}10^{-27}$ kg = 931.50 MeV/c^2]

Radius of Nucleus

$r = r_0 A^{1/3}$ where $r_o = 1.2\text{x}10^{-15}$ m = 1.2 fm

Strong Nuclear Force

- attractive force between nucleons (n-n, n-p, p-p)
- short-ranged
- very strong (must overcome p-p repulsion)

Binding Energy

$$E_b = \Delta m \cdot c^2$$

where

mass deficit: $\Delta m = Zm_{H-1} + (A - Z)m_n - m_{atom}$

Example:
Find the binding energy of a C-14 nucleus and the binding energy per nucleon.

Ans. 105.2856 MeV and 7.52 MeV per nucleon

5.2 Radioactivity

Types of Nuclear Radiation

α	2 p^+ + 2 n bound together (He-4 nucleus)
β^-	electron
β^+	positron (antielectron)
γ	gamma ray photon (sometimes emitted after α or β decay from excited daughter)

Alpha Decay

Big nucleus ejects alpha.

Decay Equation:

$${}^{A}_{Z}Parent \rightarrow {}^{A-4}_{Z-2}Daughter + {}^{4}_{2}\mathrm{He}$$

Energy released in decay:

$$Q = \Delta m \cdot c^2$$

where mass deficit $\Delta m = m_{parent} - m_{daughter} - m_{He-4}$

If gamma decay follows, part of this energy is the gamma ray photon energy with the remaining part going to kinetic energy of alpha. The gamma ray comes from the excited alpha daughter (indicated with the * in the decay equations).

$${}^{A}_{Z}Parent \rightarrow {}^{A-4}_{Z-2}Daughter^* + {}^{4}_{2}\mathrm{He}$$

$${}^{A-4}_{Z-2}Daughter^* \rightarrow {}^{A-4}_{Z-2}Daughter + \gamma$$

Example:

(a) Write the alpha and gamma decay equations for Ra-226.
(b) Find the total energy released in the decay . If the gamma ray emitted has a wavelength of 0.0067 nm, then how much kinetic energy is left for the alpha particle?

Ans. (a) ${}^{226}_{88}\mathrm{Ra} \rightarrow {}^{222}_{86}\mathrm{Rn}^* + {}^{4}_{2}\mathrm{He}$ then ${}^{222}_{86}\mathrm{Rn}^* \rightarrow {}^{222}_{86}\mathrm{Rn} + \gamma$
(b) Q = 4.869 MeV with 4.683 MeV of kinetic energy

Beta-Minus Decay

Inside the nucleus

$$\mathrm{n} \rightarrow \mathrm{p}^+ + \mathrm{e}^- + \bar{\nu}$$

where e^- is electron (β^-) and $\bar{\nu}$ is antineutrino.

Decay Equation:

$${}^{A}_{Z}Parent \rightarrow {}_{Z+1}^{A}Daughter + {}_{-1}^{0}\beta + \bar{\nu}$$

Energy released in decay:

$$Q = \Delta m \cdot c^2$$

where mass deficit $\Delta m = m_{parent} - m_{daughter}$

If gamma decay follows, part of this energy is the gamma ray photon energy with the remaining part going to kinetic energy of beta. The gamma ray comes from the excited alpha daughter (indicated with the * in the decay equations).

$${}^{A}_{Z}Parent \rightarrow {}_{Z+1}^{A}Daughter^* + {}_{-1}^{0}\beta + \bar{\nu}$$

$${}_{Z+1}^{A}Daughter^* \rightarrow {}_{Z+1}^{A}Daughter + \gamma$$

Example:
Write the beta decay equation for C-14 and find the energy released in the decay.

Ans. ${}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}N + {}_{-1}^{0}\beta + \bar{\nu}$ and Q = 0.16 MeV

Beta-Plus Decay

Only occurs for neutron-deficient nuclei.

Inside the nucleus

$$\mathrm{p}^{+} + Energy \rightarrow \mathrm{n} + \mathrm{e}^{+} + \nu$$

where e^+ is positron (β^+) and is ν neutrino.

Decay Equation:

$${}_{Z}^{A}Parent \rightarrow {}_{Z-1}^{A}Daughter + {}_{+1}^{0}\beta + \nu$$

Example:
Write the beta decay equation for N-13.

Ans. ${}_{7}^{13}\mathrm{N} \rightarrow {}_{6}^{14}C + {}_{+1}^{0}\beta + \nu$

Electron Capture (EC)

Another way for neutron-deficient nucleus to decay.

An electron is captured by the nucleus from the inner shell of the atom and combines with a proton to form a neutron and neutrino. The neutrino is ejected.

$$\mathrm{p}^{+} + e^{-} \rightarrow \mathrm{n} + \nu$$

Population Decay

$$N(t) = N_o \mathrm{e}^{-\lambda t}$$

N_o = number of unstable nuclei at $t = 0$ [#]

$N(t)$ = number of unstable nuclei at time t [#]

λ = decay constant [1/s]

Half-Life ($T_{1/2}$)

At $t = T_{1/2}$, the population reduces by one-half. $N(T_{1/2}) = N_o / 2$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Activity

The rate of decay is the activity.

$$R(t) = \lambda\, N(t)$$

$$R(t) = R_o \mathrm{e}^{-\lambda t}$$

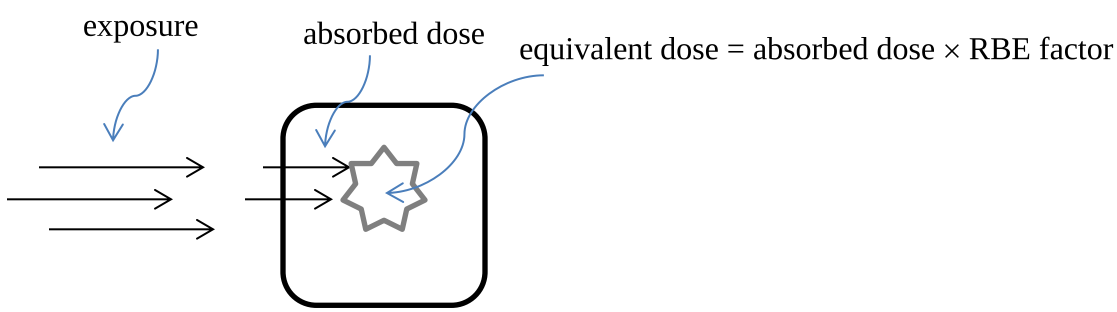
Units of activity:	mks	[decay/s = Becquerel = Bq]
	common	[Curie = Ci]
		[1 Ci = $3.7 \mathrm{x} 10^{10}$ Bq]

Probability of Decay

The probability that nuclei will decay in the next unit of time t is

$$P(t) = 1 - \mathrm{e}^{-\lambda t} = 1 - \mathrm{e}^{-0.693/T_{1/2}}$$

Health Effects



RBE (relative biological effectiveness) factor is a number that depends on type and energy of radiation.

Typical values:

gamma	1
beta	1-2
alpha	10-20

Name	Description	mks unit	common unit	conversion
Exposure	amount of ionizing radiation exposed to	[C/kg]	[Roentgen = R]	[1 C/kg = 3876 R]
Absorbed Dose	amount of ionizing radiation absorbed by tissues	[Gray = Gy]	[rad]	[1 Gy = 100 rads]
Equivalent Dose	absorbed dose times RBE factor	[Sievert = Sv]	[rem]	[1 Sv = 100 rems]

The equivalent dose and the duration of the dose are the most important quantities in determining health risk.

Average annual natural background level ~ 310 mrem
(2/3 from thoron and radon, 1/3 from other terrestrial sources, cosmic rays, internal isotopes)

Average annual artificial background level ~ 310 mrem
(mainly from medical procedures)

Annual recommended limits beyond background:
100 mrem general public, 5000 mrem occupational

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from U.S. Nuclear Regulatory Commission
http://www.nrc.gov/reading-rm/doc-collections/fact-sheets/bio-effects-radiation.html

5.3 Nuclear Power

Nuclear power reactors utilize fission of U-235.

Review the “Nuclear Physics Study Questions” posted on the course page for more information.