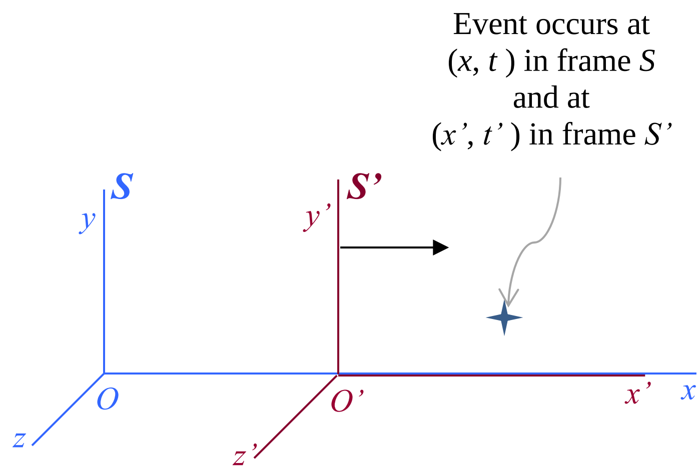
Part 4: Special Theory of Relativity

4.1. Length and Time

Lorentz Transformation Equations

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - vx/c^2)$$



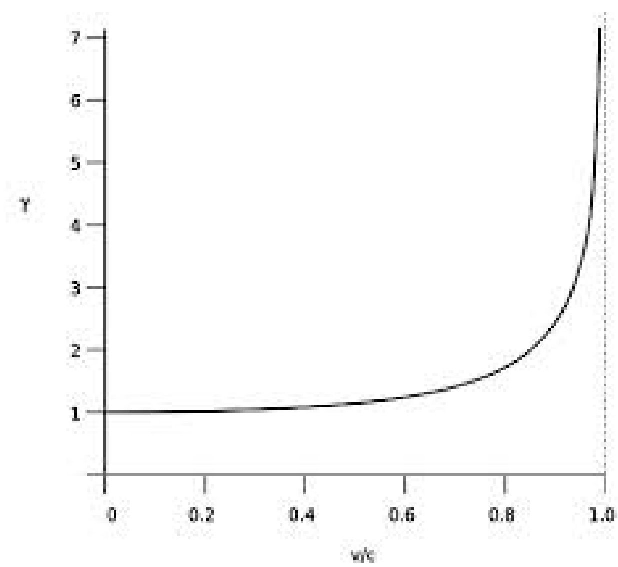
Inverse Equations

$$x = \gamma(x' + vt')$$
$$y = y'$$
$$z = z'$$
$$t = \gamma(t' + vx'/c^2)$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$v = 0 \quad \gamma = 0$

$v \to c \quad \gamma \to \infty$



Principle of Time Dilation

proper time interval Δt_p = time between 2 events that occur at the **same position**

non-proper time interval Δt_{np} = time between two events that occur at different **positions**

$$\Delta t_{np} = \gamma \cdot \Delta t_p$$

Principle of Length Contraction

To calculate a valid length, the end positions of an object must be measured at the same time if the object is moving. If the object is not moving, the end positions can be measured at different times.

proper length L_p = length of object when it is at rest

non-proper length L_{np} = length of object when it is moving

$$L_{np} = \frac{L_p}{\gamma}$$

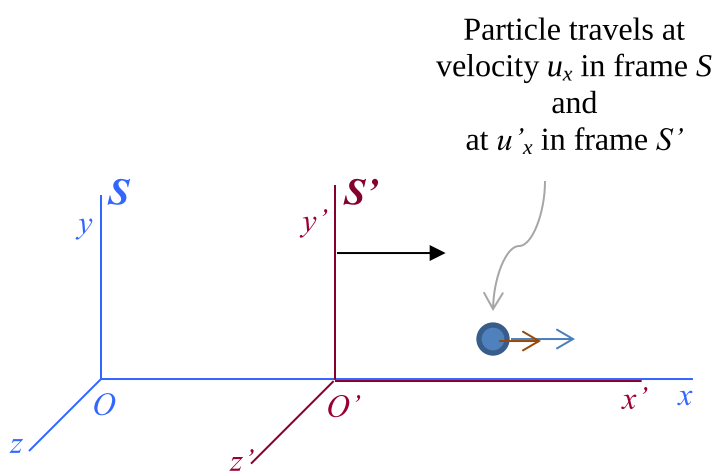
4.2 Velocity

Velocity Transform

$$u'_x = \frac{u_x - v}{1 - v u_x / c^2}$$

Inverse Transform

$$u_x = \frac{u'_x + v}{1 + v u'_x / c^2}$$

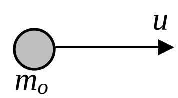


Example:
A spaceship moves away from Earth at 0.9*c* relative to Earth. It fires missile in the direction of its motion at 0.95*c* relative to the ship. It fires a second missile at 0.95*c* towards Earth relative to the ship. (a) Find the speed and directions of the two missiles relative to Earth. (b) The ship then fires a laser pulse at *c* relative to the ship in the direction of Earth. How fast does the pulse travel as measured on Earth?

Ans. (a) Missile 1 travels at 0.997*c* away from Earth, Missile 2 at 0.345*c* towards Earth (b) *c*

4.3 Momentum & Energy

Momentum



$$p = \gamma m_o u = mu$$

m_o = rest mass

$m = \gamma m_o$ = relativistic mass

$$\gamma = \frac{1}{\sqrt{1-u^2/c^2}}$$

Kinetic Energy

$$K = (\gamma - 1)m_o c^2$$

Rest Energy

$m_o c^2$

Total Energy

$$E = K + m_o c^2 = \gamma m_o c^2 = mc^2$$

Units of Energy, Mass, and Momentum

Relativistic particles are so small that the large mks units are unwieldy. We therefore use much smaller units that are based upon the energy unit of the mega-electronvolt (MeV). Notice that momentum has the same dimensions as energy divided by the speed of light. We, therefore, can express momentum in units of MeV/c. Notice that mass has the same dimensions as energy divided by the square of the speed of light. We, therefore, can express mass in units of MeV/c^2. The rest masses of common particles are listed below.

Quantity	mks unit	relativistic unit
Energy	J	MeV
Mass	kg	MeV/c^2
momentum	kg-m/s	MeV/c

particle	Rest mass (MeV/c^2)
electron	0.511
proton	938.272
neutron	939.565
photon	0

Example:
An electron moves at 0.5*c*. Find its (a) momentum, (b) kinetic energy, and (c) total energy.

Ans. (a) 0.295 MeV/*c* (b) 0.0792 MeV (c) 0.5902 MeV

Example:
An electron moves at 0.5*c*. (a) How much energy is needed to increase its speed to 0.9*c*? (b) How much more energy is need to go from 0.9*c* to 0.99*c*? (c) How much energy is needed for the electron to reach a speed of *c*?

Ans. (a) 0.58 MeV (b) 2.45 MeV (c) infinite amount…it can't be done

Effective Mass of Photon

The rest mass of a photon is zero.

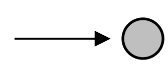
Recall that the momentum of a photon is $p = E_{ph} / c = h / \lambda$.

You can write the momentum of a photon as $p = m_{eff}\, c$ where m_{eff} is the effective mass of the photon.

Equating the expressions for momentum gives

$$m_{eff} = \frac{E_{ph}}{c^2}$$

Acceleration of Particle Acted Upon by Constant Force



$$a = \frac{F_o}{\gamma^3 m_o}$$

Note that as the particle speeds up, $u \to c$ but $\gamma \to 0$ and $a \to 0$. The particle cannot reach *c*.

An Example of Mater-Energy Equivalence: Pair Annihilation

An antiparticle is a particle that has the same rest mass as its regular particle counterpart but the opposite charge. We don't see many antiparticles in our universe because when an antiparticle meets its regular particle counterpart, their combined matter is converted into a gamma ray burst (matter is converted into energy).

Consider a particle and its antiparticle at rest. Two gamma rays with the same energy are produced in the annihilation which travel in opposite directions to conserve momentum.

Wavelength of gamma ray photon:

$$\lambda = \frac{hc}{m_o c^2} = \frac{1243 \text{ eV·nm}}{m_o c^2}$$