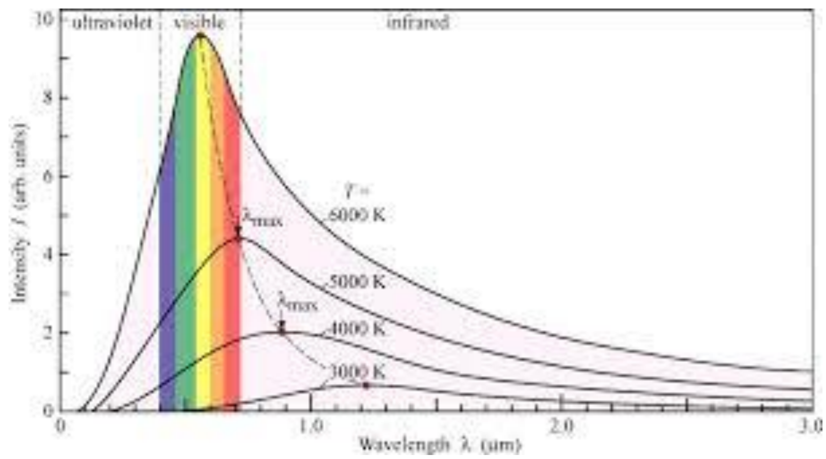
Part 3: Quantum Physics

3.1. Thermal Radiation and Photons



Temperature Conversions

$T_K = T_C + 273.15$ $T_F = (9/5)T_C + 32$

Wien's Displacement Law

$$\lambda_{max} = \frac{0.0029 \text{ m·K}}{T}$$

Total Power Emitted (Stefan's Law)

$$P_{emit} = \varepsilon\sigma A T^4$$

Total Power Absorbed from Surroundings

$$P_{abs} = \varepsilon\sigma A T_{sur}^4$$

ε = emissivity of object's surface [0-1]
σ = Stefan-Boltzmann constant = $5.67\text{x}10^{-8}$ W/(m^2-K^4)
A = surface area of object [m^2]
T = temperature of object's surface [K]
T_{sur} = temperature of surroundings [K]

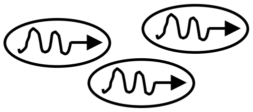
Photons

Energy of photon

$$E_{ph} = hf = \frac{hc}{\lambda_o} = \frac{1243 \text{ ev·nm}}{\lambda_o}$$

h = Planck's constant = $6.63\text{x}10^{-34}$ J-s = $4.14\text{x}10^{-15}$ eV-s
f = frequency of photon
λ_o = free-space wavelength of photon

<u>Power</u> in beam of photons



$$P = E_{ph} R_{ph}$$

where R_{ph} = photon rate [# / s]

<u>Momentum</u> of photon

$$p_{ph} = \frac{E_{ph}}{c} = \frac{h}{\lambda_o}$$

Example:
A He-Ne laer emits 7 mW of light at a wavelength of 632.8 nm. Find the (a) energy of an emitted photon, (b) the number of photons leaving the laser each second, and (c) the momentum of a photon.

Ans. (a) 1.96 eV = 3.13×10^{-19} J (b) 2.2×10^{16} (c) 1.96 ev/c = 1.045×10^{-27} kg-m/s

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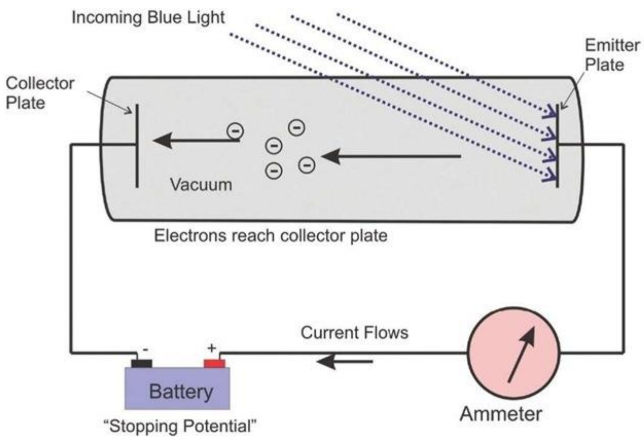
3.2. Photoelectric Effect

Electron ejected from material when struck by photon.



<u>Maximum Kinetic Energy</u> of photoelectron

$$K_{max} = hf - \phi$$

where ϕ is <u>work function</u> (binding energy) of photocathode material.

<u>Stopping Potential</u>

$$V_s = \frac{K_{max}}{e}$$

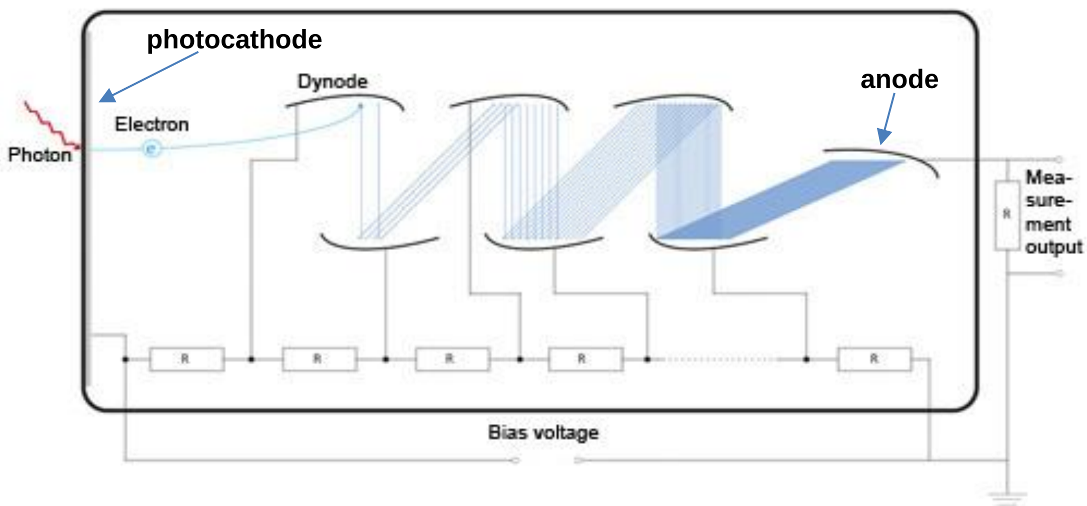
where e = fundamental charge = 1.6×10^{-19} C.

<u>Cut-off Frequency & Wavelength</u>

$$f_{co} = \frac{\phi}{h} \quad \text{and} \quad \lambda_{co} = \frac{hc}{\phi} = \frac{1243 \text{ ev}\cdot\text{nm}}{\phi}$$

To have photoelectrons ejected, $f > f_{co}$ which means $\lambda < \lambda_{co}$.

<u>Application: Photomultiplier Tube (PMT)</u>



R_{ph} = photon rate striking the PMT [#/s]
η = photocathode efficiency = probabilty that photon causes ejection of electron [#]
N = number of dynodes [#]
δ = secondary emission ratio [#]

G = gain = δ^N [#]

$R_{cathode}$ = rate of photoelectrons leaving photocathode = ηR_{ph} [#/s]

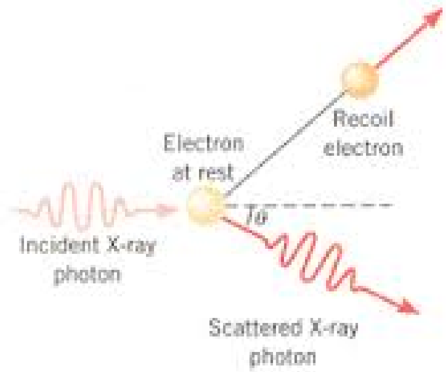
R_{anode} = rate of electrons collected at anode = $G R_{cathode}$ [#/s]

$i_{cathode}$ = current leaving cathode = $e R_{cathode}$ [A]

i_{anode} = current leaving anode = $e R_{anode}$ [A]

3.3 Compton Scattering

Scattering of x-rays off of electrons.



$$\lambda' - \lambda_o = \frac{h}{mc}(1 - \cos\theta)$$

λ_o = incident x-ray wavelength
λ' = scattered x-ray wavelength
m = mass of electron
θ = scattering angle

Compton wavelength $\lambda_C = h / mc = 0.00243$ nm

3.4 Bohr Model of Hydrogen

Postulates

1. Circular orbits via the electric (Coulomb) force.
2. Only certain stable orbits exist with distinct atomic energies.
3. Radiation in the form of a photon is emitted when the electron makes a transition from a higher orbit to a lower orbit. Radiation in the form of a photon can be absorbed and the electron makes a transition from a lower orbit to a higher orbit.
4. These stable orbits are determined by a quantized orbital angular momentum ($L = mvr$) according to

$$L = n\frac{h}{2\pi} = n\hbar$$

where the quantum number $n = 1,2,3,\ldots$ and $\hbar = h / (2\pi)$ (pronounced "h-bar")

Radii of stable orbits (quantum states)

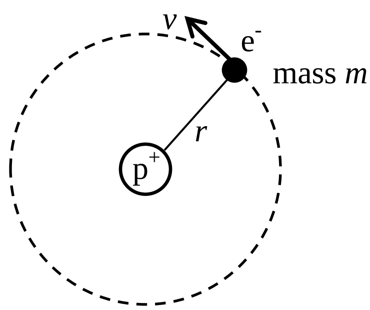
$$r_n = n^2 a_o$$

where quantum number $n = 1,2,3,\ldots$

and a_o is the Bohr radius

$$a_o = \frac{\hbar^2}{mke^2} = 0.53\ \text{Å}$$

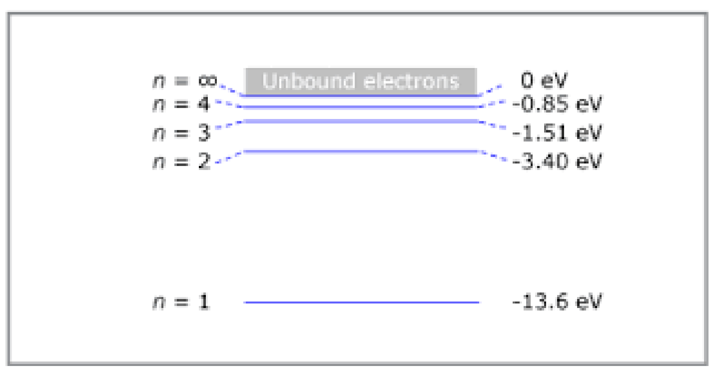
k = Coulomb's constant
e = fundamental charge



Energies of quantum states

$$E_n = -\frac{ke^2}{n^2 a_o} = -\frac{13.6\ \text{eV}}{n^2}$$

$n = 1$ ground state
$n = 2$ first excited state
$n = 3$ second excited state
etc.



A photon is emitted when the atom makes a transition from a higher level to a lower level.
A photon is absorbed when the atom makes a transition from a lower level to a higher level.

The energy of that photon is the difference of the atomic energies of those levels.

$$E_{ph} = \left|E_{nf} - E_{ni}\right|$$

where E_{ni} and E_{nf} are the initial and final atomic energies, respectively.

The wavelength of that photon can then be found using

$$\lambda_o = \frac{1243 \text{ ev}\cdot\text{nm}}{E_{ph}}$$

Example:

(a) Find the wavelength of the photon emitted when a Hydrogen atom makes a transition from the second excited state to the first excited state. In what part of the spectrum is the photon?

(b) Find the wavelength of the photon absorbed if a Hydrogen atom goes from the ground state to the first excited. In what part of the spectrum is the photon?

Ans. (a) 656 nm, red (b122 nm, uv

3.5 Theory of Quantum Mechanics

Matter Waves (deBroglie Postulates)

Wavelength and frequency of particle as a wave:

$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad f = \frac{E}{h}$$

Example:

(a) Find the wavelength of a softball (mass of 8 ounces) travelling at 40 mph. Is there any way to detect this wavelength?
(b) Find the wavelength of an electron moving at 1000 km/s. Is there any way to detect this wavelength?

Ans. (a) $1.6\text{x}10^{-34}$ m No, wavelngth is too small. (b) 0.73 nm Yes.

Heisenberg Uncertainty Principle

Position-Momentum:

Suppose a particle is moving in one dimension and you measure its position and momentum at the same time.
The position measurement has an uncertainty Δx so that the particle's position is $x \pm \Delta x$.
The momentum measurement has an uncertainty Δp so that the particle's momentum is $p \pm \Delta p$.

Because the particle acts as a wave,

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

Energy-Time:

Suppose you measure the energy of a particle and the time that it has that energy.
The energy measurement has an uncertainty ΔE so that the particle's energy is $E \pm \Delta E$.
The time measurement has an uncertainty Δt so that the time is $t \pm \Delta t$.

Because the particle acts as a wave,

$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$$

Example:

The speed of an electron is measured to be 10^4 m/s with an uncertainty of 0.01%. The position of the electron is measured at the same time. What is the smallest uncertainty in the position measurement that quantum mechanics allows?

Ans. ±0.12 mm

1-D Wave Function

$\Psi(x,t)$ is the wave function that describes the particle as a wave with the following properties.

- Probability of finding the particle between x and $x+dx$ at time t = $|\Psi(x,t)|^2\, dx$
- Probability of finding the particle between a and b at time t = $\int_a^b |\Psi(x,t)|^2\, dx$
- Normalization Condition: $\int_{-\infty}^{\infty} |\Psi(x,t)|^2\, dx = 1$
- $\Psi(x,t)$ and $\frac{\partial \Psi}{\partial x}$ must be continuous and single-valued
- $\Psi(x,t) \to 0$ as $x \to \pm\infty$

1-D Time-Independent Schroedinger Equation

If the wave function can be written as the product of a spatial function and a temporal function, then the system can be analyzed independent of time by solving the 1-D time-independent Schroedinger Equation. That is, if

$$\Psi(x,t) = \psi(x)\cdot\varphi(t)$$

then

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + U(x)\psi(x) = E\psi(x)$$

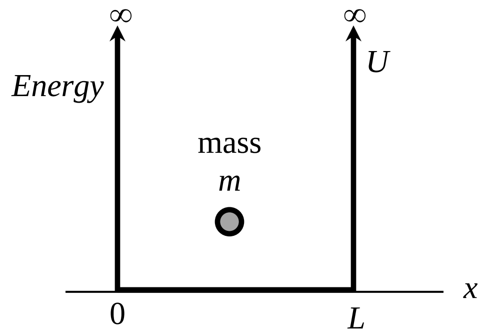
← 1-D time-independent Schroedinger Equation

where $U(x)$ is the potential energy of the particle and E is the total energy of the particle.

There are only certain values of E and certain eigenfunctions $\psi(x)$ that are solutions to this differential equation. The particle's energies are "quantized".

Example: 1-D Infinite Square Well

For a particle trapped in a layer, the potential energy can be approximated as



$$U(x) = \begin{cases} 0 & 0 < x < L \\ \infty & \text{else} \end{cases}$$

The solutions to the Schroedinger Equation are

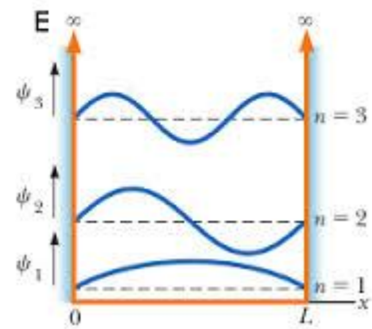
$$\psi(x) = A\sin k_n x$$

$$k_n = n\frac{\pi}{L}$$

$$E_n = \frac{\hbar^2 k_n^2}{2m} = n^2\frac{\pi^2\hbar^2}{2mL^2}$$

where the quantum number n = 1,2,3,…

The matter wave in a particular state forms a standing wave with wavelength



$$\lambda_n = \frac{2\pi}{k_n} = \frac{2L}{n}$$

3.6 Atomic Physics

Quantum Numbers
Quantum mechanics applied to atoms give rise to the following five quantum numbers. (The spin-orbit magnetic coupling of the electron is ignored.)

Symbol	Name	What it quantizes	How it quantizes	Allowed values
n	principle	energy E	$E = -\left(\frac{Z_{eff}}{n}\right)^2 (13.6\text{ eV})$ where Z_{eff} depends on n and l	$n = 1, 2, 3,\ldots$
l	orbital	orbital angular momentum L	$L = \sqrt{l(l+1)}\hbar$	$l = 0, 1,\ldots, n-1$
m_l	orbital magnetic	z-component of orbital angular momentum L_z	$L_z = m_l\hbar$	$m_l = -l, -l+1, .., 0, .., l$
s	spin	spin angular momentum S	$S = \sqrt{s(s+1)}\hbar$	$s = 1/2$
m_s	spin magnetic	z-component of spin angular momentum S_z	$S_z = m_s\hbar$	$m_s = -1/2$ or $+1/2$

Note 1: Since the spin quantum number is always ½, you only need the remaining four quantum numbers to describe an atomic state.

Note 2: The two possible values of the spin magnetic quantum number are often referred to as "spin up" for +1/2 and "spin down" for -1/2.

Note 3: The range of orbital l values depends on the principle quantum number n. The orbital quantum number ranges from 0 to n-1 in integer steps.

Note 4: The range of orbital magnetic m_l values depends on the orbital quantum number l. The orbital magnetic quantum number ranges from $-l$ to $+l$ in integer steps.

A quantum state is given by a set of quantum numbers of the form (n, l, m_l, m_s).

For a given n, there are $2n^2$ atomic states.

Example:
How many atomic states are there for $n = 2$? List the states by their quantum numbers.

Ans. 8
(2,0,0,+½) (2,0,0,- ½) (2,1,-1,+ ½) (2,1,-1,- ½) (2,1,0,+ ½) (2,1,0,- ½) (2,1,1,+ ½) (2,1,1,- ½)

Chemistry (Spectroscopic) Notation

n determines the shell   K (n = 1) L (n = 2) M (n = 3)

l determines the subshell   s (l = 0) p (l = 1) d (l = 2) f (l = 3)

n, l, m_l together determine the orbital

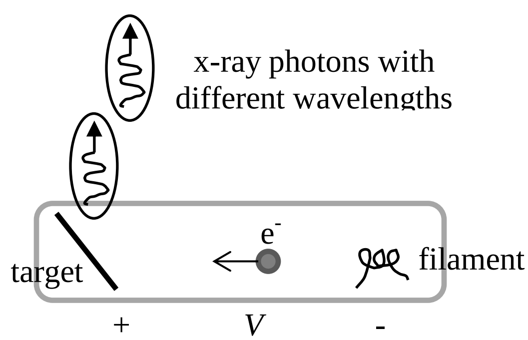
Pauli Exclusion Principle

No two electrons in an atom can have the same set of quantum numbers. (No two electrons in an atom can be in the same state.)

Note that the exclusion principle implies the filling of orbitals, subshells, and shells that we see in chemistry and the structure of the periodic table.

X-Ray Production

$$\lambda_{min} = \frac{hc}{eV}$$



Electrons are ejected from the K-shell (n=1) of atoms in the collisions. The K characteristic lines are photons emitted from electron transitions from higher shells to the vacant K-shells. K_α is a transition from the L-shell ($n = 2 \rightarrow 1$). K_β is a transition from the M-shell ($n = 3 \rightarrow 1$).

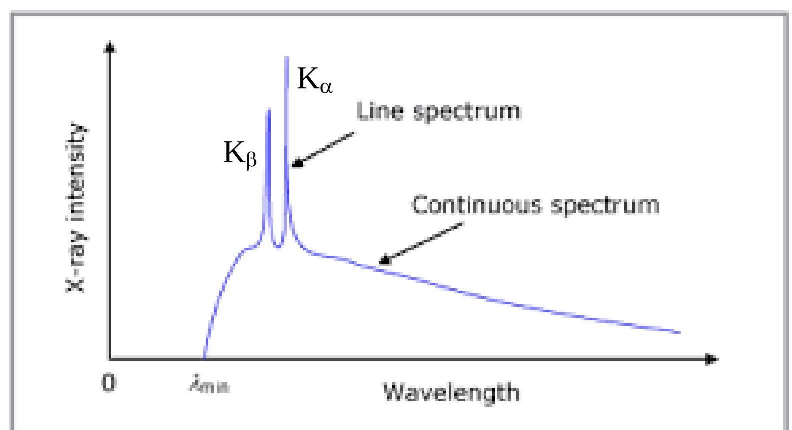


Figure 3

Lasers

Review the Powerpoint slides on "Lasers" located on the course page:
http:/facstaff.cbu.edu/~jvarrian/252/252LasersWeb/252Lasers.ppt