

## Part 1: Light & Geometric Optics

### 1.1: Electromagnetic Waves

The electric and magnetic field waves are coupled together in the electromagnetic wave. The fields oscillate in perpendicular directions and the wave travels in a direction perpendicular to each field's oscillation axis. For a wave traveling along the  $x$ -axis in a non-conducting medium, the fields obey the wave equations of the form

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{\epsilon\mu} \frac{\partial^2 E}{\partial t^2}$$

and

$$\frac{\partial^2 B}{\partial x^2} = \frac{1}{\epsilon\mu} \frac{\partial^2 B}{\partial t^2}$$

where  $\epsilon$  and  $\mu$  are the electrical permittivity and magnetic permeability of the medium, respectively.

In a harmonic wave, the fields can be written using a cosine or sine function.

$$E(x, t) = E_M \frac{\cos}{\sin}(kx \pm \omega t + \varphi) \quad \text{and} \quad B(x, t) = B_M \frac{\cos}{\sin}(kx \pm \omega t + \varphi)$$

$E_M$  = electric field amplitude of wave [V/m]

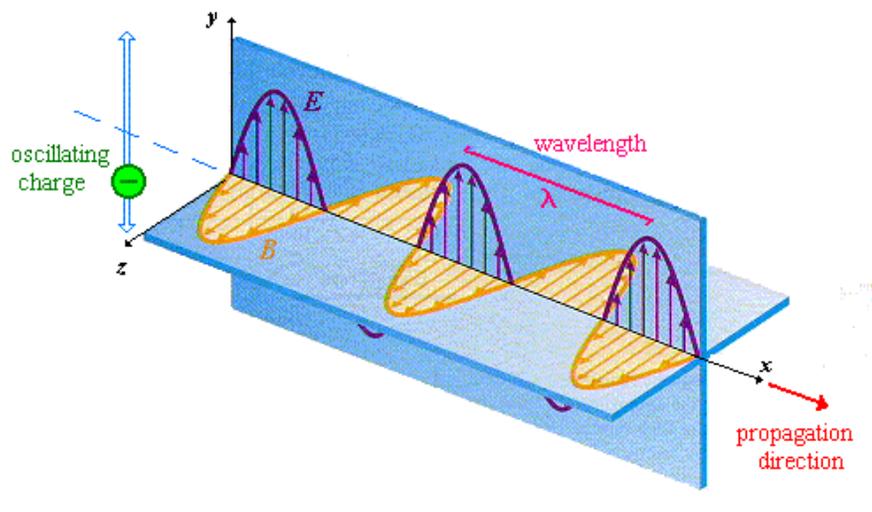
$B_M$  = magnetic field amplitude of wave [T]

$k$  = angular wave number [rad/m]

$\omega$  = angular frequency [rad/s]

$\varphi$  = phase angle [rad]

A harmonic electromagnetic wave traveling in the  $+x$  direction is shown below.



$\lambda$  = wavelength = distance between consecutive crests [m]

$k = 2\pi / \lambda$  [rad/m]

$T$  = period = time for one oscillation or cycle [s]

$f = 1 / T$  = frequency = number of cycles per time [cycles/s = Hz]

$\omega = 2\pi / T = 2\pi f$  [rad/s]

The  $\pm$  sign between  $kx$  and  $\omega t$  determines the direction the wave travels along the  $x$ -axis.

+ wave travels to left (in the direction of decreasing  $x$ )

- wave travels to right (in the direction of increasing  $x$ )

The phase angle  $\phi$  shifts the cosine or sine function left or right. This can be used to match some initial condition for the wave function.

Speed of the harmonic wave (phase velocity)

$$v = \frac{E_M}{B_M} = \lambda f = \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}}$$

In free space (vacuum),

$$v = \frac{E_M}{B_M} = \lambda f = \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon_0\mu_0}} = c = 3 \times 10^8 \text{ m/s}$$

This is the fastest that light can travel and only light can travel at this speed. In any other medium, the speed of light is

$$v = \frac{c}{n}$$

where  $n$  is the refractive index or index of refraction.

For vacuum,  $n = 1$ . For any other medium,  $n > 1$ . Air's index is approximately 1.

Average intensity [W/m<sup>2</sup>] carried by wave in vacuum

$$I_{ave} = \frac{E_M B_M}{2\mu_0} = \frac{E_M^2}{2\mu_0 c} = \frac{c B_M^2}{2\mu_0}$$

## Electromagnetic Spectrum

Frequency of light determines the part of the spectrum. You should know the order of the spectrum and of the colors in the visible part of the spectrum (ROYGBV).

## Light in Different Media

Frequency is constant, speed and wavelength change.

$$n_1 \lambda_1 = n_2 \lambda_2$$

Light in vacuum has a free-space wavelength of  $\lambda_0$ . Wavelength in medium with index  $n$  is

$$\lambda = \lambda_0/n$$

*Example:*

A laser pointer delivers a red beam in air with a wavelength of 650 nm. What is the wavelength and color of the beam underwater ( $n \sim 1.33$ )?

*Ans.* 488 nm and the beam remains red because the frequency is the same

## **1.2 Reflection & Refraction**

### Law of Reflection

$$\theta_1 = \theta_r$$

### Law of Refraction (Snell's Law)

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

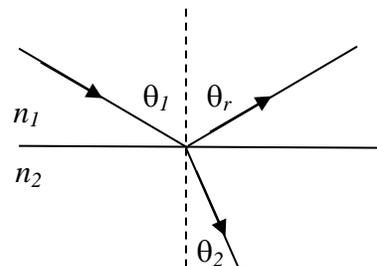
Case 1:  $n_1 < n_2$  (lo index into hi index)  $\theta_1 > \theta_2$  (light bends towards normal like in picture)

Case 2:  $n_1 > n_2$  (hi index into lo index)  $\theta_1 < \theta_2$  (light bends away from normal)

Critical Angle (only for Case 2)

$$\sin \theta_c = \frac{n_2}{n_1}$$

If  $\theta_1 > \theta_c$  then all light reflects for Total Internal Reflection (TIR)



### Normal Dispersion

Index decreases as wavelength increases.

For light going from air into medium, violet bends most, red least.

### 1.3 Image Formation

$p$  = object distance (from object to element)

$q$  = image distance (from element to image)

$h$  = object height (from axis to tip)

$h'$  = image height (from axis to tip)

#### Lateral Magnification

$$M = \frac{h'}{h}$$

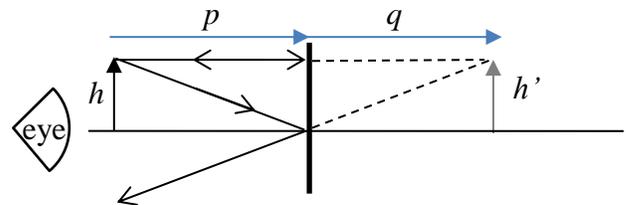
#### Plane Mirror

$q = p$

$h' = h$

$M = 1$

virtual, non-inverted image



#### Single Refracting Surface (Apparent Depth)

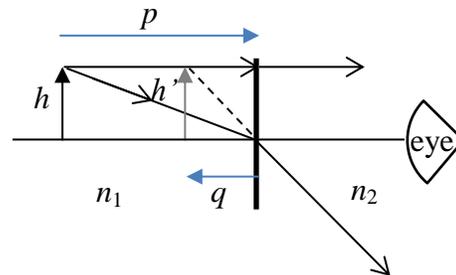
$|q| < p$

$h' = h$

$M = 1$

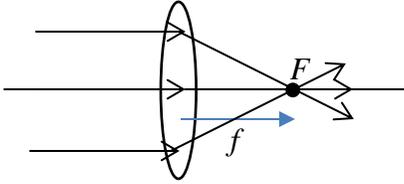
virtual, non-inverted image closer to surface

for small angles,  $|q| \approx \frac{n_2}{n_1} p$

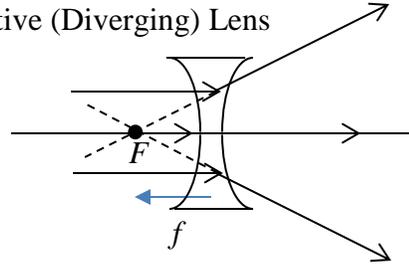


## Thin Lens

Positive (Converging) Lens



Negative (Diverging) Lens



### Lens Maker's Equation

$$\frac{1}{f} = \left( \frac{n_{lens} - n_{sur}}{n_{sur}} \right) \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

$F$  = focal point

$f$  = focal length

$n_{lens}$  = index of lens material

$n_{sur}$  = index of medium surrounding lens

$R_1$  = radius of curvature of left surface (front surface)

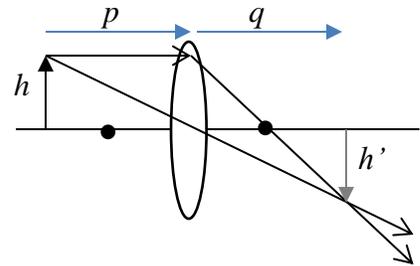
$R_2$  = radius of curvature of right surface (back surface)

$R > 0$  for ( -shaped surface

$R < 0$  for ) -shaped surface

### Thin Lens Equation

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$



### Lateral Magnification

$$M = \frac{h'}{h} = \frac{-q}{p}$$

## 1.4 Simple Optical Systems

(You should know how to do the ray traces for these systems.)

### Camera

- positive lens, object outside focal point and far from lens
- real, inverted image
- image smaller than object

### Projector

- positive lens, object outside focal point but close to lens
- real, inverted image
- image bigger than object

### Magnifier (Magnifying Glass)

- positive lens, object inside focal point
- virtual, non-inverted image
- image bigger than object

### Angular Magnification

$$m = \frac{25 \text{ cm}}{p}$$

Largest useful angular mag occurs when  $q = -25 \text{ cm}$ . Then, and only then,  $m = M$ .

$$m_{max} = \frac{25 \text{ cm}}{f} + 1$$

### *Example:*

You use a lens with a focal length of +5 cm as a magnifying glass to look at a 1 cm-long bug.

- How far is the lens from the bug if you get the maximum useful angular mag?
- What is the maximum useful mag?
- What is the lateral mag?
- How long does the bug look to you?

Suppose you now move the lens so that it is 4.8 cm from the bug.

- How far is the image of the bug from you?
- What is the angular mag now?
- What is the lateral mag now?
- How long does the bug look to you?

*Ans.* (a) 4.17 cm (b) 6x (c) 6x (d) 6 cm (e) 120 cm (f) 5.2x (g) 25x (h) 5.2 cm

## Eye: Correcting Vision Problems with Lenses

### Myopia (Near-Sightedness)

- can't see far-away objects
- eye has an unhealthy far point (F.P.) of less than infinity
- need negative lens to fix

$$f = -\text{F.P.}$$

### Hyperopia (Far-Sightedness)

- can't see near objects
- eye has an unhealthy near point (N.P.) greater than 25 cm
- need positive lens to fix with  $p = 25 \text{ cm}$  and  $q = -\text{N.P.}$

$$\frac{1}{f} = \frac{1}{25 \text{ cm}} - \frac{1}{\text{N.P.}}$$

### Lens Power (L.P.)

$$L.P. = \frac{1}{f}$$

mks units [1/m = diopter]