

Intrinsic Carrier Concentration

Number of conduction electrons per volume: $n = N/V = \int D_c(\epsilon) f_c(\epsilon) d\epsilon / V$

where $D_c(\epsilon)$ is the density of possible states, and $f_c(\epsilon)$ is the fraction of each state of energy ϵ that is filled:

Density of states: $D_c(\epsilon) \equiv dN/d\epsilon = (V/2\pi^2) (2m_e/\hbar^2)^{3/2} \epsilon^{1/2}$ (from Part 3, Free Electron Fermi Gas, section 5),

where for free electron: $\epsilon_{free} = \hbar^2 k^2 / 2m_e$;

for conduction electron: $\epsilon = E_{at\ bottom\ of\ conduction\ band} + \hbar^2 k^2 / 2m_e^*$,

or relative to the top of the valence band: $\epsilon = E_{gap} + \hbar^2 k^2 / 2m_e^*$ so $\epsilon_{free}^* = \hbar^2 k^2 / 2m_e^* = (\epsilon - E_{gap})$

therefore: $D_c(\epsilon) = (V/2\pi^2) (2m_e^*/\hbar^2)^{3/2} (\epsilon - E_{gap})^{1/2}$ for $\epsilon \geq E_{gap}$.

Fermi-Dirac distribution: $f_c(\epsilon) = 1 / [e^{(\epsilon-\mu)/kT} + 1]$

The Fermi-Dirac distribution is for a fixed number of particles and where the Pauli Exclusion principle only allows either zero or one particle per state. Here the k in the exponent is the Boltzmann Constant, k_B ; and μ is the Fermi level, sometimes called the chemical potential, which is the energy for which $f = 1/2$. Note: the Fermi level is not the same as the Fermi energy (which is the energy of the top filled level at temperature 0. See Appendix on Distribution Functions under Part 2.

If we have a small enough Temperature: $k_B T \ll (\epsilon - \mu)$,

note: at room temperature, $k_B T \approx .02$ eV; as we'll see later, $\mu \approx \epsilon_{gap} \approx 0.2$ eV

then $e^{(\epsilon-\mu)/kT} \gg 1$, so (neglecting the 1 in the denominator) :

$$f_c(\epsilon) \approx 1 / e^{(\epsilon-\mu)/kT} = e^{-(\epsilon-\mu)/kT} = e^{-\epsilon/kT} e^{+\mu/kT} .$$

Therefore putting all this back together gives:

$$n = N/V \approx (1/V) (V/2\pi^2) (2m_e^*/\hbar^2)^{3/2} \int_{E_{gap}}^{\infty} (\epsilon - E_{gap})^{1/2} e^{-\epsilon/kT} d\epsilon e^{+\mu/kT} .$$

To make the \int go from 0 to ∞ , let's make the substitution:

$$x = (\epsilon - E_{gap}) / k_B T, \quad dx = d\epsilon / k_B T, \quad \text{and} \quad \epsilon / k_B T = x + E_{gap} / k_B T;$$

also, when $\epsilon = E_{gap}$ (lower limit of \int), $x=0$; when $\epsilon = \infty$, $x = \infty$.

$$\text{Therefore: } n \approx (1/2\pi^2) (2m_e^*/\hbar^2)^{3/2} (k_B T)^{3/2} e^{+\mu/kT} \int_0^{\infty} x^{1/2} e^{-x} dx e^{-E_{gap}/kT} ,$$

but the integral is a definite one and from tables:

$$\int_0^{\infty} x^{1/2} e^{-x} dx = 1/2(\pi)^{1/2} .$$

$$\text{Finally then: } n \approx 2(m_e^* k_B T / 2\pi \hbar^2)^{3/2} e^{(\mu - E_{gap})/kT} .$$

To determine n we need to know: m_e^* , T , μ , and E_{gap} . We can get m_e^* from cyclotron resonance; we can get E_{gap} from optical absorption.

It is also useful to calculate the **density of holes, p**:

$$p = N/V = \int D_h(\epsilon) f_h(\epsilon) d\epsilon / V .$$

1) If we keep the same reference of $\epsilon=0$ at top of valence band, limits on \int are from $-\infty$ to 0;

$$2) \text{ Density of states: } D_h(\epsilon) \equiv dN/d\epsilon = (V/2\pi^2) (2m_h/\hbar^2)^{3/2} (-\epsilon)^{1/2} ,$$

since $\epsilon_c = E_{top\ of\ valence\ band} - \hbar^2 k^2 / 2m_h = -\hbar^2 k^2 / 2m_h$;

$$3) f_h = 1 - f_c = 1 - 1/[e^{(\epsilon-\mu)/kT} + 1] \quad (\text{fraction of holes, } f_h, \text{ is simply the fraction of "missing" electrons!)$$

$$= \{ [e^{(\epsilon-\mu)/kT} + 1] / [e^{(\epsilon-\mu)/kT} + 1] \} - \{ 1 / [e^{(\epsilon-\mu)/kT} + 1] \}$$

$$= e^{(\epsilon-\mu)/kT} / [e^{(\epsilon-\mu)/kT} + 1] = 1 / [1 + e^{-(\epsilon-\mu)/kT}] \approx e^{+(\epsilon-\mu)/kT} = e^{\epsilon/kT} e^{-\mu/kT}$$

[\approx comes from assuming $k_B T \ll (\mu - \epsilon)$; recall $\epsilon < 0$ so $(\mu - \epsilon)$ always $> \mu$]

Therefore: $p \approx (1/V) (V/2\pi^2) (2m_h^*/\hbar^2)^{3/2} \int_{-\infty}^0 (-\varepsilon)^{1/2} e^{\varepsilon/kT}] e^{-\mu/kT}$

again to get a definite integral let $x = -\varepsilon/k_B T$, $\varepsilon = -k_B T x$:

$$p \approx (1/2\pi^2) (2m_h^*/\hbar^2)^{3/2} e^{-\mu/kT} (k_B T)^{3/2} \int_0^{\infty} x^{1/2} e^{-x} dx]$$

As before, the integral $\int_0^{\infty} x^{1/2} e^{-x} dx = 1/2\pi^{1/2}$,

so we finally get: $p \approx 2(m_h^* k_B T/2\pi\hbar^2)^{3/2} e^{-\mu/kT}$.

compare to: $n \approx 2(m_e^* k_B T/2\pi\hbar^2)^{3/2} e^{(\mu-E_{gap})/kT}$.

See the excel plot: [Intrinsic concentration versus temperature.](#)

Law of Mass Action:

Consider $n * p \approx \{2(m_e^* k_B T/2\pi\hbar^2)^{3/2} e^{(\mu-E_{gap})/kT}\} * \{2(m_h^* k_B T/2\pi\hbar^2)^{3/2} e^{-\mu/kT}\}$

or $n p \approx 4(k_B T/2\pi\hbar^2)^3 (m_e^* m_h^*)^{3/2} e^{-E_{gap}/kT}$. (law of Mass Action)

Note: This does NOT involve μ

except that we have assumed $|\varepsilon - \mu| \gg k_B T$ where ε is measured from top of filled valence band; $k_B T = 0.026$ eV at $T = 300$ K (room temperature).

Homework Problem 33 deals with this.

E_{gap} values are in Table 1 of Chapter 8 in Kittel, 6th ed. and are generally in the range 0.3 to 3 eV.

Note: **for constant T, $n p = \text{constant}$.** That is, if we add impurity we may increase n ; but this implies that if n goes up, p goes down so that $n p = \text{constant}$. This is called **compensation**. If we have more electrons than holes (see the next section), then some of those extra electrons will fall down and “cancel” some of the holes thus decreasing the number of holes. However, total carrier concentration is $n + p$.

Homework Problem 34 deals with this aspect.

Example: if n goes up by 1%, then $n' = 1.01 n$.

But $n p = n' p'$, so $p' = (n/n') p = (1/1.01) p$;

thus $n' + p' = 1.01 n + (1/1.01) p$;

but intrinsically, $n = p$, so $n + p = 2n$, or $n = 1/2(n + p)$,

so $n' + p' = (1.01 + [1/1.01]) 1/2(n + p) = 1.00005 (n + p)$!

Chemical Potential (or Fermi Level), μ

In semiconductor physics, μ is called the Fermi level. Elsewhere, μ is sometimes called the chemical potential.

Note that the Fermi level is a little bit different than the Fermi energy, although at $T=0$ we have $\mu = \epsilon_F$.

Recall that $f(\epsilon) = 1/[e^{(\epsilon-\mu)/kT} + 1]$, so that $f(\epsilon=\mu) = 1/[e^0 + 1] = 1/2$.

For an **intrinsic** semiconductor, $n = p = (np)^{1/2}$.

We now use the law of mass action for np above $np \approx 4(k_B T/2\pi\hbar^2)^3 (m_e^* m_h^*)^{3/2} e^{-E_{gap}/kT}$ to get:

$$n_{intrinsic} = (np)^{1/2} = 2(k_B T/2\pi\hbar^2)^{3/2} (m_e^* m_h^*)^{3/4} e^{-E_{gap}/2kT}.$$

From our previous equation for n we got:

$$n = 2(m_e^* k_B T/2\pi\hbar^2)^{3/2} e^{(\mu - E_{gap})/kT}.$$

Now setting the two expressions equal gives: $2(k_B T/2\pi\hbar^2)^{3/2} (m_e^* m_h^*)^{3/4} e^{-E_{gap}/2kT} = 2(m_e^* k_B T/2\pi\hbar^2)^{3/2} e^{(\mu - E_{gap})/kT}$

$$e^{\mu/kT} = (m_h^*/m_e^*)^{3/4} e^{E_{gap}/2kT}, \quad \text{or}$$

$$\mu/k_B T = (3/4) \ln[m_h^*/m_e^*] + E_{gap}/2k_B T, \quad \text{or}$$

$$\mu = 1/2 E_{gap} + (3/4) k_B T \ln(m_h^*/m_e^*).$$

Thus, if $m_h^* \approx m_e^*$, then $\mu = 1/2 E_{gap}$ for intrinsic semiconductors. This will be important when we consider how doped semiconductors "fit" together to make diodes and transistors!

Mobility - in the intrinsic range

(We are also going to use the symbol μ for mobility as well as for the Fermi level! So be careful of the context.)

$$\text{mobility: } \mu_x \equiv |v_x| / E_x \quad (\text{where } E_x \text{ is the electric field in the } x \text{ direction})$$

(In ideal intrinsic semiconductors, the mobility is determined by collisions between electrons and phonons.)

Recall that $j = \sigma E$ AND $j = n(-e)v$.

Therefore $\sigma = j/E = n(-e)v/E = ne\mu$ (actually, $\sigma = ne\mu_e + np\mu_h$).

Recall that $v = qE\tau/m$ [from $F=ma$ which is $qE = m(\delta v/\tau)$ where δv is the extra speed that causes the current]

so that $\mu = v/E$, or $\mu_e = e\tau_e/m_e$ and $\mu_h = e\tau_h/m_h$.

(τ depends on the number of phonons and so is a function of T , but as a power law, not as an exponential like n and p .)

Recall from the Planck distribution: $n_{avg} = 1/[e^{\epsilon/kT} - 1]$, so for $\epsilon \ll k_B T$ we have $n_{avg} \approx 1/[1 + \epsilon/k_B T - 1] = k_B T/\epsilon$, where $\epsilon = \hbar\Omega \approx 0.1 k_B T$ at room T – see homework problems 12 and 16)

UNITS of μ are $(m/s)/(V/m) = m^2/(V\cdot s)$ [or $cm^2/V\cdot s$]

Here are a few values of carrier mobilities at room temperature, in $cm^2/V\cdot s$

Crystal	electrons	holes
Si	1,350	480
Ge	3,600	1,800
GaAs	8,000	300

compare to value of μ for Cu = 35 $cm^2/V\cdot s$.

Semiconductors have high μ compared to metals since small gaps (small U 's) imply small effective masses.

(Recall from the previous section: $m_e^* = m_e/[1 - 2\epsilon_{edge}/U]$.)