Comparisons between electrons and phonons

Phonons	Electrons
Starting place: Newton's 2nd Law	Starting place: Schrodinger's Equation
Waves of oscillating atoms	Waves of probability of finding electrons
Boundary conditions set $K = 2\pi/\lambda = n\pi/L$	Boundary conditions set $k = 2\pi/\lambda = n2\pi/L$

n is quantum number in both
both energy and momentum are quantized in both

Phonons	Electrons
Discrete nature of atoms limits $K<\pi/a$	Continuous nature of e wave does not limit k but boundary conditions do limit k; potential energy of atoms is periodic and so affects electrons with k values near $n2\pi/a$
$p = \hbar K$	$p = \hbar k$
$E_{lattice} = E_{phonon} = \hbar\Omega = hf$	$\varepsilon_{free\ electron} = \frac{1}{2} mv^2 = \hbar^2k^2/2m$
$v_{phase} = \Omega/K \approx 10^3$ m/s	$v_F = \hbar k_F/m_e$ where $k_F = [3\pi^2N/V]^{1/3} \approx 10^6$ m/s
Planck distribution can create phonons can have more than one in any particular energy level $\langle n\rangle = 1 / [e^{\varepsilon/kT} - 1]$	Fermi-Dirac distribution can't create electrons can't have more than one in any energy level $f_{Fermi\text{-}Dirac}(\varepsilon) = 1 / [e^{(\varepsilon-\mu)/kT} + 1]$

Heat Capacity: classical theory: $C = (3/2)Nk_B$

Phonons	Electrons
At T=0, still some oscillations due to $E_{HO} = (n+ \frac{1}{2})\,\hbar\Omega$	At T=0, still some energy due to filling up of energy levels up to ε_F
small T: $C_{ph} = 234Nk_B(T/\Theta)^3$ where $\Theta = \hbar\Omega_D/k_B$ and $\Omega_D = [6\pi^2v^3N/V]^{1/3}$	small T: $C_{el} = \frac{1}{2}\pi^2Nk_B(T/T_F)$ where $T_F = \varepsilon_F/k_B$ and $\varepsilon_F = (\hbar^2/2m)(3\pi^2N/V)^{2/3}$
Θ (Debye Temp) ranges from 200 to 2,000 K	T_F ranges from 20,000 to 120,000 K

Thermal conductivity: $K_T = (1/3)ncv^2\tau = (1/3)(nc)v(v\tau) = (1/3)Cv\,l$

Photons

Starting place: Maxwell's Equations

Waves of oscillating electromagnetic fields

Continuous nature of E&M fields does not limit k

$p = \hbar k$ $\quad E_{photon} = \hbar\omega = hf$

$v_{phase} = c$ (in vacuum) $= 3 \times 10^8$ m/s or $v = c/n$ (in material) $\approx 10^8$ m/s
{here n = index of refraction of the material}