Thermal Conductivity of Metals

1. Thermal Conductivity

Homework problem 28 asks you to derive the following expression for the thermal conductivity due to the electrons:

$$\mathbf{K_{el} = \tfrac{1}{3} C_{el} v \mathit{l}},$$

where C_{el} is the heat capacity of the electrons, v is the speed of the electrons, and l is the mean free path of the electrons. [HINT: This derivation is similar to the one performed earlier for phonons.]

From the heat capacity section, we obtained an expression for C_{el} per volume:

$$\mathbf{C_{el} = \tfrac{1}{2}\pi^2 (N/V) k_B{}^2 T/\varepsilon_F}$$

where $\varepsilon_F = \tfrac{1}{2} m v_F{}^2$ was the Fermi energy. Most of the energy and hence most of the heat will be carried by those electrons near the Fermi level, and hence **to a very good approximation we can use the Fermi velocity for the v in the thermal conductivity**. Likewise, the mean free path is simply the distance travelled during the time between collisions, and this should be, to a very good approximation, $\mathbf{v_F \tau \approx \mathit{l}}$. Therefore:

$$K_{el} = \tfrac{1}{3}[\tfrac{1}{2}\pi^2 (N/V) k_B{}^2 T / \tfrac{1}{2} m v_F{}^2]\,[v_F]\,[v_F\tau] = \mathbf{\pi^2 (N/V) k_B{}^2 \tau T / 3m}.$$

Note that **τ does depend on T** since τ depends on the number of phonons which depends on T. Specifically, as T goes up, the number of phonons goes up, and so τ goes down.

Recall that the thermal conductivity due to the phonons was:

$$\mathbf{K_{phonons} = \tfrac{1}{3} C_{lat} v \mathit{l}}$$

where $C_{lat} \approx 3Nk_B/V$ at high temperatures and $C_{lat} \approx 234Nk_B(T/\Theta)^3$ at low temperatures, v was speed of sound, and l was the mean free path $\approx v\tau$.

In general then, the thermal conductivity of metals is going to be dominated by the electronic contribution (mainly because the electrons are going to be moving a lot faster than the phonons).

For low T, $C_{lat}/C_{el} = [234(N/V)k_B(T/\Theta)^3]/[\tfrac{1}{2}\pi^2(N/V)k_B{}^2T/k_BT_F] = (468/\pi^2)(T/\Theta)^3/(T/T_F)$; $v_{phonon} \approx 10^3$ m/s, $v_{Fermi} \approx 10^6$ m/s.

For higher T, $C_{lat}/C_{el} = [3(N/V)k_B]/[\tfrac{1}{2}\pi^2(N/V)k_BT/T_F] = (6/\pi^2)T_F/T \approx 100$; $v_{Fermi}/v_{phonon} \approx 1{,}000$.

2. Ratio of Thermal to Electrical Conductivity

The ratio of thermal to electrical conductivity is (with n=N/V):

$$K/\sigma = [\pi^2 n k_B{}^2 \tau_K T/3m]\,/\,[ne^2\tau_\sigma/m] = (\pi^2 k_B{}^2/3e^2)\,T = (2.45\text{x}10^{-8}\ \text{Watt-Ohm/deg}^2)\,T.$$

Note that this ratio is independent of the material! A good electrical conducting material will also be a good thermal conducting material, and an electrical insulating material will also be a thermal insulating material. This ratio has its own name: the **Wiedemann-Franz law**: the ratio of the thermal to the electrical conductivity is proportional to the temperature with the value of the proportionality constant being independent of material. The above assumes that τ_K and τ_σ are about the same. At room temperature we get very good agreement. At $T \approx 15$K we have $\tau_K \approx (1/10)\tau_\sigma$.

To get the value of the proportionality constant, which we call the Lorenz number, L, we simply take:

$$L \equiv K/\sigma T = \pi^2 k_B{}^2/3e^2 = 2.45 \times 10^{-8}\ \text{Watt-Ohm/deg}^2.$$

Homework problem #29 is about this situation.