BOLTZMANN DISTRIBUTION

Probability of one atom having n units of energy is based on equal likelyhood of any state. Below is a listing of all possible states for two cases.

CASE I: four atoms having three units of energy:

	ABCD		ABCD	ABCD	ABCD	ABCD		ABCD
(3000) 4	3000	(2100) 12	2100	1200	1020	1002	(1110) 4	1110
	0300		2010	0210	0120	0102		1101
	0030		2001	0201	0021	0012		1011
	0003							0111

Prob of atom A having:

P(3) = 1/20 = .05
P(2) = 3/20 = .15
P(1) = 6/20 = .30
P(0) = 10/20 = .50

CASE II: four atoms having five units of energy:

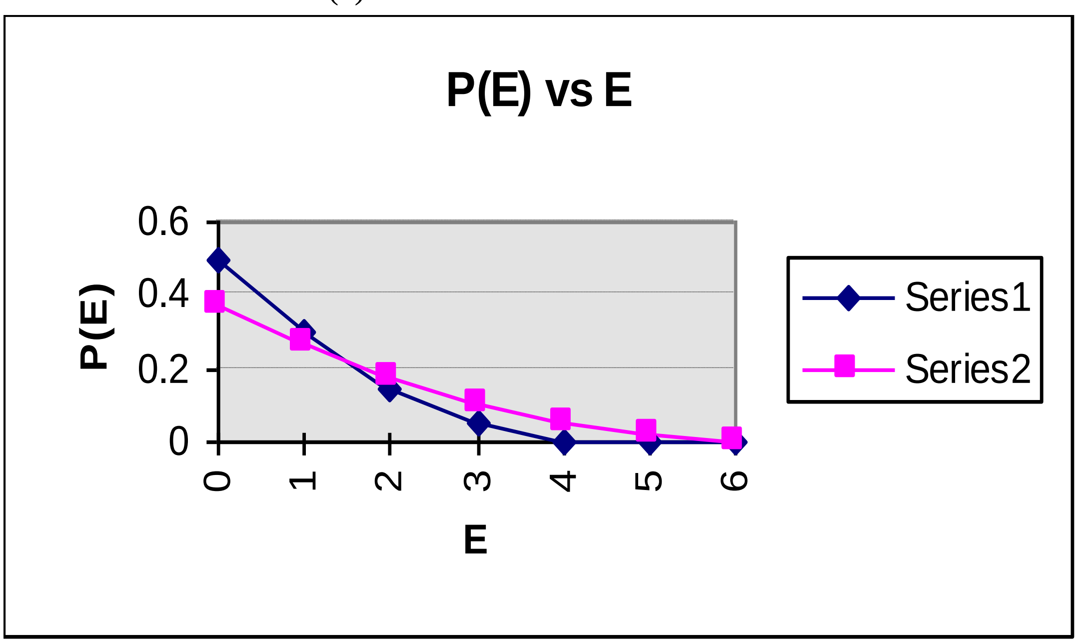
	ABCD		ABCD	ABCD	ABCD	ABCD		ABCD
(5000) 4	5000	(4100) 12	4100	1400	1040	1004	(2111) 4	2111
	0500		4010	0410	0140	0104		1211
	0050		4001	0401	0041	0014		1121
	0005							1112

	ABCD	ABCD	ABCD	ABCD		ABCD	ABCD	ABCD	ABCD
(3200)12	3200	2300	2030	2003	(3110) 12	3110	1310	1130	1103
	3020	0320	0230	0203		3101	1301	1031	1013
	3002	0032	0032	0023		3011	0311	0131	0113

	ABCD	ABCD	ABCD	ABCD	ABCD	ABCD
(2210)12	2210	2120	2102	1220	1202	1022
	2201	2021	2012	0221	0212	0122

Prob of atom A having:

P(5) = 1/56 = .018
P(4) = 3/56 = .054
P(3) = 6/56 = .107
P(2) = 10/56 = .179
P(1) = 15/56 = .268
P(0) = 21/56 = .375



As you can see from the above graph, the probability of having a certain amount energy gets smaller as the amount of energy increases. This appears to look like a **dying exponential**.

Also as you can see, as the amount of available energy increases, the curve still looks like a dying exponential, but the probability of having zero energy is less than when there was less total energy, and the probability of having a higher energy increases as the available energy increases. The total probability (area under the curve) still adds to the same total probability of 1.

If we make the average energy = $k_B T$, then the average energy is just the sum of their individual energies times their probabilities: $E_{avg} = k_B T = \Sigma(n\varepsilon)Ae^{-\alpha n\varepsilon}/\Sigma Ae^{-\alpha n\varepsilon} \rightarrow \int(n\varepsilon)Ae^{-\alpha n\varepsilon}d\varepsilon / \int Ae^{-\alpha n\varepsilon}d\varepsilon = 1/\alpha$ (by integrating the numerator by parts), so $\alpha = 1/(k_B T)$. Thus the **Boltzmann distribution** function is $\mathbf{P(n\varepsilon) = Ae^{-n\varepsilon/kT}}$.

Compare this to the Binomial Distribution: The probability distribution, called the **Binomial Distribution**, that says each of 4 atom has a ¼ chance of getting each unit of energy allows the chance that the four atoms will accumulate more or less than the total amount of 3 units of energy available.

Binomial Probability of A not getting a particular unit of energy is (3/4).

The probability of A not getting any of the 3 units is $(3/4)^3$.

The probability of all four atoms not getting any of the units of energy is then $[(3/4)^3]^4 = (.75)^{12} = .03 \neq 0$.

The Boltzmann distribution says that we must distribute the 4 units of energy, so the probability of all four atoms not getting any units of energy has to be zero, but the Binomial distribution, as seen above, does have a non-zero probability of have all four atoms having zero.