# Taylor Series Review

$$f(x-x_o) = f(x_o) + (df/dx)|_{xo}*(x-x_o)/1! + (d^2f/dx^2)|_{xo}*(x-x_o)^2/2! + ...$$

Taylor Series are usually good approximations whenever $(x-x_o)$ is small compared to 1.

**Example #1:** $f(x) = \sqrt{(1+x)} = (1+x)^{½}$ ; $x_o = 0$

$$f(x) = (1+0)^{½} + ½(1+0)^{-½}\,x + -¼(1+0)^{-3/2}\,x^2/2 + ...$$

or

$$f(x) = \sqrt{[1+x]} = 1 + ½x - (1/8)x^2 + ...$$

for x = 0.5, we get f(x) = 1.2247449 (from calculator)
f(x) = 1 + .250 - .03125 + ... ≈ 1.2187500 (from first three terms)
NOTE: this is not all that close since 0.5 is not « 1

for x = 0.1, we get f(x) = 1.0488088 (from calculator)
f(x) = 1 + .05 - .00125 + ... ≈ 1.0487500 (from first three terms)
NOTE: this is a much better approximation

for x = 0.01, we get f(x) = 1.0049876 (from calculator)
f(x) = 1 + .005 - .0000125 + ... ≈ 1.0049875 (from first three terms)
NOTE: this is a better approximation still!

**Example #2:** $f(x) = e^{+\alpha x}$; $x_o = 0$

$$f(x) = e^0 + \alpha e^0\,x + \alpha^2 e^0\,x^2/2 + ...$$

or

$$f(x) = e^{+\alpha x} = 1 + \alpha x + ½(\alpha x)^2 + ...$$

for αx = 0.1, we get f(x) = 1.1051709 (from calculator)
f(x) = 1 + .1 + .005 + ... ≈ 1.1050000 (from first three terms)

**Example #3:** $f(\theta) = A[1 - \cos(\theta)]$; $\theta_o = 0$

$$f(\theta) = A[1 - \cos(0)] + A\sin(0)\,\theta + A\cos(0)\,\theta^2/2 + ...$$

or

$$f(\theta) = [1 - \cos(\theta)] = 0 + 0 + \theta^2/2 + ...$$

for θ = 0.5° and A = 1, we get f(θ) = 0.0000381 (from calculator)
f(θ) = 0 + 0 + .125 + ... ≈ 0.1250000 (from first 3 terms)
NOTE: this is not very good - this does NOT work for degrees!

for θ = 0.5° = .0087266 radians f(θ) = 0.0000381 (from calculator)
f(θ) = 0 + 0 + .0000381 + ... ≈ 0.0000381 (from first 3 terms)
for θ = 10° = .1745329 radians f(θ) = 0.0151922 (from calculator)
f(θ) = 0 + 0 + .0152309 + ... ≈ 0.0152309 (from first 3 terms)

Other useful power series expansions that we will use later are:

**$\sin(\theta) = \theta$** - …

**$\cos(\theta) = 1 - \frac{1}{2}\theta^2$ + …**

**Caution:** the argument of sine and cosine, $\theta$, must be in radians – not degrees – for the approximation to be valid.

HINT: to check your approximation, try putting in a small value for the argument and seeing the result on your calculator.

For example, $e^{0.01} = 1.01005\ldots$ which is $1 + .01 + (.01)^2/2 +$ which is $1 + (kx) + (kx)^2/2 +$ .

**Inverting the process**

After using some of the more common ones, you can recognize the series as belonging to a particular function, and hence you can then find the value of the series by finding the value of the function!