

# STUDY GUIDE FOR PART 4: SPECIAL RELATIVITY

## Special Theory of Relativity

### A. The speed of light & the Michelson-Morley experiment

Q-1 to 5;

1. inertial frames
2. speed relative to what?
3. Michelson-Morley experiment

### B. Transformation Equations

Q-14; S-63 to 67

0. Galilean Transformation equations (one event):  $x_B = (x_A \pm vt_A)$ ;  $t_B = t_A$

1. Lorentz transformation equations (one event)

$$x_B = (x_A \pm vt_A)/\sqrt{[1-v^2/c^2]}; t_B = (t_A \pm [v/c^2]x_A)/\sqrt{[1-v^2/c^2]}$$

2. velocity transformation equations

$$u_B = (u_A \pm v)/(1 \pm u_A v/c^2)$$

3. determine which sign to use in the LTE equations

### C. Measurements in different frames

Q-6,7,13; S-68,69,70

1. time dilation (time differences) [need one frame to use ONE clock]

$$\Delta t_{np} = \Delta t_{proper}/\sqrt{[1-v^2/c^2]}, \Delta t_{np} \text{ uses two clocks; } \Delta t_{proper} \text{ uses one}$$

2. length contraction (length differences) [need object at REST in one frame]

$$\Delta L_{np} = \Delta L_{proper} \times \sqrt{[1-v^2/c^2]}, \Delta L_{np} \text{ is moving; } \Delta L_{proper} \text{ is stationary}$$

3. mass measurement [need object at REST in one frame]

$$m_{np} = m_{proper}/\sqrt{[1-v^2/c^2]}, m_{np} \text{ is moving; } m_{proper} \text{ is stationary}$$

### D. Mass and Energy

Q-8 to 12; S-71 to 77

1. kinetic energy  $KE = m_{np}c^2 - m_{proper}c^2 = m_{proper}c^2 \times [(1/\sqrt{[1-v^2/c^2]}) - 1]$

2. total energy  $E_{total} = m_{np}c^2 = m_{proper}c^2 + KE$

### E. More experiments

1. mass
  - a) cyclotron and synchrotron
  - b) nuclear decays
  - c) pair production

2. time

- a) direct measurements
- b) radioactive decay

F. General relativity: non-inertial frames

**STUDY QUESTIONS:**

- 1) What was the purpose of the Michelson-Morley experiment?
- 2) The failure of the Michelson-Morley experiment leads to what conclusion about the speed of light?
- 3) What basic assumption is made in deriving the classical transformation equations?
- 4) What is the difference between special relativity and general relativity?
- 5) Give the two postulates of special relativity.
- 6) How would measured values of length (of dimension parallel to the motion) and mass of a moving object compare with the values obtained when it is at rest?
- 7) What is meant by time dilation and length contraction?
- 8) What is rest mass energy?
- 9) How do you compute rest mass energy, total energy, and kinetic energy in the theory of special relativity?
- 10) Briefly, why is  $c$  the upper limit for the speed of a material object?
- 11) What is pair production, and why is it of special significance?
- 12) It should be clear from this section that the statement "matter can neither be created nor destroyed" is false. What can one say instead?
- 13) Explain why it is possible for a person who lives less than 100 years to travel distances from earth greater than 100 light years yet never go faster than light.

14) Why don't we observe relativistic effects in our everyday lives?

**SUPPLEMENTARY PROBLEMS (S- ):**

63) Two events 20 meters apart occur simultaneously in the unprimed reference frame. An observer in a primed reference frame moving at a speed of  $0.8c$  relative to the unprimed frame also sees the events. What time difference does the primed observer measure? What spatial separation does the primed observer measure?

64) Two events that occur 10 km apart are separated by  $30 \mu\text{s}$ . The later event is further out in the  $+x$  direction. How fast and in what direction must an observer travel in order to see them as simultaneous? How far apart would the events be for this observer?

65) A lady on Venus sees two space ships approaching her from opposite directions at speeds of  $0.9c$  and  $0.8c$ , respectively. What is the relative speed of the two ships as measured by an observer on either of them?

66) According to observers on earth, a rocket moves east at a speed of  $0.5c$  directly toward a photon moving west at a speed  $c$  along the same line. What is the speed of the photon according to observers on the rocket?

67) (a) With respect to the laboratory, ball A rolls eastward with a constant speed of  $2 \text{ m/sec}$  and ball B rolls westward with a constant speed of  $2 \text{ m/sec}$ . What is the relative velocity of A with respect to B, calculated relativistically? (b) With respect to the laboratory, the electrons in accelerator A are projected toward the east with a speed of  $2 \times 10^8 \text{ m/sec}$  and electrons in accelerator B are projected toward the west with a speed of  $2 \times 10^8 \text{ m/sec}$ . What is the relative velocity of the A electrons with respect to the B electrons?

68) A certain process requires  $1.00 \mu\text{s}$  to occur in an atom at rest in the lab. How much time will this process require when the atom is moving at a speed of  $0.6c$  ?

69) How fast would a space ship have to go relative to an observer on the earth for each year on the ship to correspond to two years on the earth?

70) How fast would a space ship have to go relative to an observer on earth for its length to be contracted to  $0.6$  of its length measured at rest on the earth?

71) How many joules of energy per kilogram of rest mass are required to accelerate a space ship to a speed of  $0.9c$  ?

72) A particle moves at a speed such that its kinetic energy just equals its rest mass energy. How fast is the particle moving?

73) An electron in a certain x-ray tube is accelerated from rest through a potential difference of 180,000 volts in going from the cathode to the anode. When it arrives at the anode what is its (a) kinetic energy in eV? (b) its relativistic mass? (c) its relativistic velocity? (d) the value of  $e/m$ ? (e) its velocity calculated classically?

74) Calculate the amount of work in MeV that must be done (a) to bring an electron from rest to a velocity of  $0.4c$ ; and (b) to increase its velocity from  $0.4c$  to  $0.8c$ . (c) What is the ratio of the kinetic energy of the electron at the velocity of  $0.8c$  to that at  $0.4c$  when computed from (1) relativistic values and (2) from classical values?

75) Show that the rest mass of an electron is equivalent to  $0.511$  MeV.

76) The fissioning of an atom of  $U^{235}$  releases  $200$  MeV of energy. What percent is this fission energy of the total which would have been available if all the mass of the uranium atom had appeared as energy?

77) A gamma ray photon having a wavelength of  $4.5 \times 10^{-13}$  m materializes into an electron-positron pair in the neighborhood of a heavy nucleus. What is the total kinetic energy of the pair in MeV immediately after being produced?

#### ANSWERS TO SUPPLEMENTARY PROBLEMS:

63)  $-89$  ns [- means event farther from origin is observed first),  $33.3$  m

64)  $0.9c$  in  $+x$  direction;  $4.37$  km.      65)  $0.988c$ .      66)  $c$

67) a)  $4.00$  m/sec; b)  $2.77 \times 10^8$  m/s.      68)  $1.25 \mu s$ .      69)  $0.866c$ .

70)  $0.8c$ .      71)  $1.16 \times 10^{17}$  joules.

72)  $0.866c$

73) a)  $180,000$  eV; b)  $12.3 \times 10^{-31}$  kg; c)  $2.02 \times 10^8$  m/s =  $.673c$ ; d)  $1.30 \times 10^{11}$  Coul/kg;  
e)  $2.52 \times 10^8$  m/s =  $.840c$ .

74) a)  $0.047$  MeV; b)  $0.294$  MeV; c1)  $6.3$ ; c2)  $4.0$ .

76)  $0.1\%$ .      77)  $1.74$  MeV.