Study Guide for Part Five

ELECTROMAGNETIC WAVES

Wave Motion

A. Waves on a string S-40,41

1. The starting point: $\Sigma \mathbf{F} = m\,\mathbf{a}$

2. wave equation (differential equation): $T_s \partial^2 y/\partial x^2 = \mu\, \partial^2 y/\partial t^2$

3. solutions: $y(x,t) = A \sin(kx \pm \omega t) + B \cos(kx \pm \omega t) = C \sin(kx \pm \omega t + \phi)$
   where $v = \sqrt{(T_s/\mu)}$ and $\omega/k = \lambda f = v$ ($k = 2\pi/\lambda$ and $\omega = 2\pi/T_p = 2\pi f$)

4. superposition of waves & Fourier Series

B. Wave properties

1. amplitude (A,B,C above) has units of waving quantity

2. frequency ($f = \omega/2\pi = 1/T_p$)

3. phase velocity (v): speed at which any $\theta = (kx \pm \omega t - \phi)$ moves: $v = \omega/k = \lambda f$

4. energy transmission (proportional to A^2, ω^2, v)

5. polarization (direction of oscillation relative to direction of motion of wave)

   (a) longitudinal

   (b) transverse

Supplementary Problems (S-):

40) Write the general differential equation for a wave in one dimension.

41) a) Show that $y(x,t) = A \sin(kx - \omega t)$ is a solution to the one dimensional wave equation.

b) Show that this wave moves in the +x direction with a speed of ω/k.

Sound Waves

A. Transmission of sound

1. type of wave

2. velocity of sound

   a) $v = \sqrt{(B/\rho)}$ for solids/liquids; $B = -\Delta P/(\Delta V/V)$ = bulk modulus

   b) $v = \sqrt{(\gamma RT/M)}$ for gases: for air, $\gamma = 1.4$; T in Kelvins, R = gas constant

B. Intensity, Energy, Power, and Loudness levels S-42,43,44

1. Intensity = Power/area (Watts/m²); for point source: $I = P/4\pi r^2$

   a) Intensity in Watts/m²: $I_{av} = \frac{1}{2} \rho \omega^2 s_m^2 v$ (for point source: $s_m \propto r^{-1}$)

   b) Intensity in decibels: $I(db) = 10 \log(I/I_o)$ where $I_o = 10^{-12}$ W/m²

2. Pressure and displacement: $\Delta P_m = \rho v \omega s_m$

3. Human hearing

C. Doppler effects: $f_{rec} = f_{source} (v_{sound} \pm v_{rec})/(v_{sound} \pm v_{source})$ S-45

Supplementary Problems (S-):

42) What is the range of frequency that the human hear is capable of hearing? What wavelengths in air do these limiting frequencies have?

43) To maintain a constant power, what must the displacement amplitude of a speaker do if the frequency decreases by a factor of ½ ?

a) increase by a factor of 4
b) increase by a factor of 2
c) increase by a factor of √2
d) remain the same
e) decrease by a factor of (1/√2)
f) decrease by a factor of ½
g) decrease by a factor of ¼

44) a) In Watts/m², what is the intensity of a sound wave at the 83 dB level? b) In dB, what is the intensity level of a sound wave at 2.7×10^{-3} Watts/m² ?
c) If the intensity in part b were doubled to 5.4×10^{-3} W/m², what would the intensity level in dB become?
d) If the source of the sound were a point source, and if the point source moved four times further away, what would the intensity level do?

45) A train moving East at a speed of 15 m/s approached a person in a car moving West at a speed of 24 m/s.
a) If the train emits a sound of frequency 1,000 Hz, what will the car observer measure for the frequency? (Assume no wind.)
b) After the train and car pass and start heading away from each other, what will the car observer measure for the frequency? (Again assume no wind.)
c) If there is a wind of 20 m/s blowing from the East (that is, blowing toward the West which is in the opposite direction the train is heading), would the answer to part b increase, stay the same, or decrease?

Answers to Supplementary Problems:

44. a) 2×10^{-4} W/m²; b) 94.3 dB; c) 97.3 dB; d) decrease by a factor of 16 in W/m², and decrease by 12 dB.

45. a) 1118 HZ; b) 891 Hz; c) increase (897 Hz).

Superposition and Standing Waves

A. Interference

B. Standing Waves and Beats

C. Resonance S-46

Supplementary Problems (S-):

46) A string with L = 32 cm and μ = .015 g/cm is stretched with a tension of 557 Nt. What is the highest harmonic of this string that is within the typical human's audible range (up to 20,000 Hz) ?

Answers to Supplementary Problems:

46) 21st harmonic (f_o = 952 Hz)

Electromagnetic Waves

A. Maxwell's equations

1. Gauss' law for electric fields: $\oiint \mathbf{E} \bullet d\mathbf{A} = Q_{enclosed}/\varepsilon_o$
2. Gauss' law for magnetic fields: $\oiint \mathbf{B} \bullet d\mathbf{A} = 0$
3. Ampere's law: $\oint \mathbf{B} \bullet d\ell = \mu_o I + \mu_o\varepsilon_o \, d/dt \, [\iint \mathbf{E} \bullet d\mathbf{A}]$
4. Faraday's law: $\oint \mathbf{E} \bullet d\ell = -d/dt \, [\iint \mathbf{B} \bullet d\mathbf{A}]$

B, Maxwell's equations in space (Q=0, I=0) and in differential form rather than the above integral form

1. Gauss' law for electric fields: $\nabla \bullet \mathbf{E} = 0$
2. Gauss' law for magnetic fields: $\nabla \bullet \mathbf{B} = 0$
3. Ampere's law: $\nabla \times \mathbf{B} = \mu_o\varepsilon_o \, \partial \mathbf{E}/\partial t$
4. Faraday's law: $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$

C. Electromagnetic Wave equation S-47,48,49

1. wave equation: combine Ampere's law and Faraday's law to get:
   $$\partial^2 \mathbf{E}/\partial x^2 = \mu_o\varepsilon_o \partial^2 \mathbf{E}/\partial t^2$$
2. velocity of waves: $v = \sqrt{[1/(\mu_o \varepsilon_o)]} = 3 \times 10^8$ m/s = c
3. light and other types of radiation
4. human response to light

D. Energy and Momentum of radiation S-50

1. Electric field energy: from parallel plate capacitor:
   Energy = $\frac{1}{2}CV^2$ where $C = \varepsilon_o A/d$ and $V = E \bullet d$
   so Energy/Vol = $\frac{1}{2} \varepsilon_o E^2$
2. Magnetic field energy: from solenoidal inductor:
   Energy = $\frac{1}{2} L I^2$ where $L = \mu_o N^2 A / L$ and $B = \mu_o (N/L) I$
   [or $I = L B / (\mu_o N)$] so Energy/Vol = $\frac{1}{2} B^2 / \mu_o$

3. S_{avg} = energy/sec*area = $E_m^2/(2\mu_o c)$ = $B_m^2 c/2\mu_o$ ($E_m/B_m = c$)

$\mathbf{S_{inst}} = (1/\mu_o)\, \mathbf{E}\times\mathbf{B}$ [the ½ in the average above comes from average of $\sin^2(\theta)$]

4. Radiation Pressure = S/c (Pressure = Force/Area, Force = $\Delta\mathbf{p}/\Delta t$)

Supplementary Problems (S-):

47) Given the wavelength (in vacuum) of the wave, tell what type (e.g., visible, I.R., radio, etc.) and if visible, what color it is.

48) Given the frequency of the wave, tell what type it is.

49) What range of wavelengths (in air) do human eyes respond to?

50) Can you use light to "sail" with in outer space? Would a white or black sail be better? Explain your answers.