Study Guide for Part Four

Faraday's Law

A. **Faraday's law** of induction

1. magnetic flux: $\iint \mathbf{B} \bullet d\mathbf{A}$
2. emf (concept of voltage generated)
3. Faraday's law: $\Delta V = d/dt\,[\iint \mathbf{B} \bullet d\mathbf{A}] = -\oint \mathbf{E} \bullet d\ell$

B. **Lenz's law:** determine direction: S-31,32

1. If losing flux, induced current will try to replace lost flux;
2. If gaining flux, induced current will try to oppose increasing flux.

C. Applications S-33

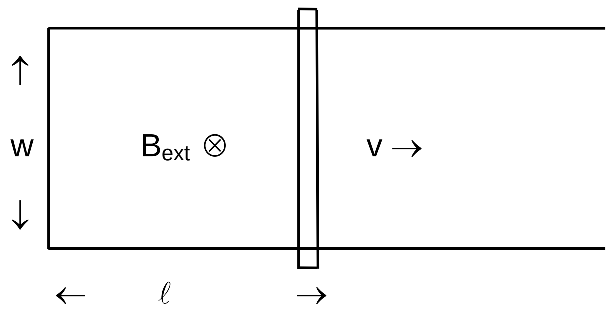
1. electric generators
2. rms vs. average vs. maximum
3. eddy currents

D. **Maxwell's equations** S-34

1. Gauss' law of electric fields: $\oiint \mathbf{E} \bullet d\mathbf{A} = Q_{enclosed}/\varepsilon_o$
2. Gauss' law of magnetic fields: $\oiint \mathbf{B} \bullet d\mathbf{A} = 0$
3. Ampere's law: $\oint \mathbf{B} \bullet d\ell = \mu_o I + \mu_o\varepsilon_o\, d/dt\,[\iint \mathbf{E} \bullet d\mathbf{A}]$
4. Faraday's law: $\oint \mathbf{E} \bullet d\ell = -d/dt\,[\iint \mathbf{B} \bullet d\mathbf{A}]$

Supplementary Problems (S-):

31) Consider the situation depicted in the figure below:

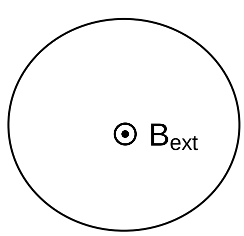


The conducting bar is moving at a speed of 4 m/s (v) to the right across conducting rails that are 10 cm apart (*w*). The present length of the bar from the connected end of the rails is 20 cm (ℓ). The strength of the external magnetic field is a constant and uniform .034 T (B_{ex}).

a) What is the voltage induced in the circuit formed by the bar and the rails?
b) If the circuit has a resistance of 5 Ω, what will the induced current be in the circuit due to this induced voltage?
c) If the speed of the bar is twice as fast, how will the induced current in the circuit change?
d) If the width of the circuit is twice as wide, how will the induced current in the circuit change?
e) If the present position of the bar is twice as far from the end of the rails, how will the induced current in the circuit change?

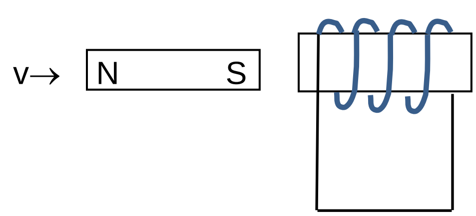
32) Clearly draw in the **direction** of the induced current in each of the four diagrams, or write NONE if there is no induced current.

a) the external magnetic field directed UP through circuit A is decreasing in strength:



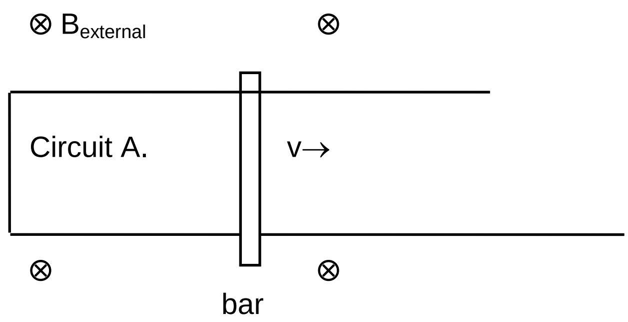
Circuit A is the loop of wire

b) the South pole of the bar magnet is pointing toward the solenoid in circuit A and the magnet is moving towards it:

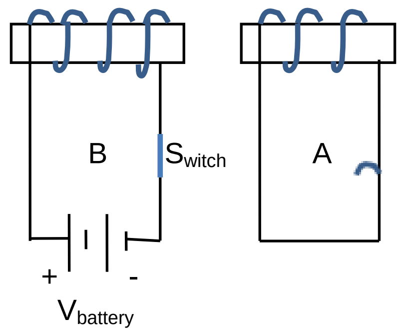


Circuit A

c) The bar is moving to the right:



d) The switch in circuit B is CLOSED (and remains closed):



33) DESIGN an electric generator that gives an rms voltage of 120 volts.

34) Know the individual names of the equations that form the basis of Maxwell's equations.

Answers to Supplementary Problems:

31) a) .0136 volts; b) 2.7 mA; c) double; d) double; e) remain the same.

Inductance

A. Mutual inductance: $V = d/dt\,[\iint \mathbf{B} \bullet d\mathbf{A}] = M\,dI/dt$ (since $B \propto I$)

units of M (and L below): Volt-sec/Amp = Henry

B. Self inductance: $V = L\,dI/dt$

C. Energy Storage: $E_{stored} = Q\,V = \int I\,V\,dt = \int I\,(L\,dI/dt)\,dt = \int L\,I\,dI = \frac{1}{2} L\,I^2$

D. Circuits with inductance S-35,36

1. RL circuits: $V_L = L\,dI/dt$; $V_R = R\,I$

$V_R = V_L$ so get $-R\,I = L\,dI/dt$, solve for I(t)

like mass with resistance: $-bv = ma$: $-b\,v = m\,dv/dt$, solve for v(t)

2. LC circuits: $V_L = L\,dI/dt$; $V_C = Q/C$ ($dI/dt = d^2Q/dt^2$)

$V_L = V_C$ so get $-L\,d^2Q/dt^2 = Q/C$, solve for Q(t); $I = dQ/dt$

get natural frequency of oscillation: $\omega = 1/\sqrt{(LC)}$

like mass on spring: $-kx = ma$: $-k\,x = m\,d^2x/dt^2$, solve for x(t), get $\omega = (k/m)^{1/2}$

3. RLC circuits: $V_R = R\,I$; $V_L = L\,dI/dt$; $V_C = Q/C$

$$V_R + V_L + V_C = 0 \quad \text{so get} \quad L\,d^2Q/dt^2 + R\,dQ/dt + Q/C = 0$$

like damped spring: $-kx-bv = ma$: $m\,d^2x/dt^2 + b\,dx/dt + k\,x = 0$

Supplementary Problems (S-):

35) Consider an inductor of 800 mH.
a) How much current is required for 1 joule of energy to be stored in the inductor?
b) If a constant DC current of 5 amps runs through the inductor, what is the voltage across the inductor?
c) If an AC current of 5 amps at a frequency of 200 Hertz runs through the inductor, what is the induced voltage across the inductor?

36) a) Design a series LRC circuit that has a natural resonance frequency of 600 kHz.
b) Design a mechanical "circuit" that has a frequency of 0.06 Hz.
c) Give the mechanical analogue for the following: applied voltage; current; inductance; capacitance; charge on the capacitor.

Answers to Supplementary Problems:

35) (a) 1.581 amps; (b) 0 volts; (c) (V_L = 5,027 volts) * sin[(1257 rad/sec)*t] or 3554 volts-rms.

36) lots of correct answers.

Alternating Current Circuits

A. Components

1. resistors $V_R = R\,I$ $X_R = R$
2. inductors $V_L = L\,dI/dt$ $X_L = \omega L$
3. capacitors $V_C = Q/C$ $X_C = 1/(\omega C)$ [$I = dQ/dt$, $Q = \int I\,dt$]

B. Circuits S-37,38

1. Impedance (Z) and phase angle (f)

a) $Z = \sqrt{[R^2 + (X_L - X_C)^2]}$

b) $\tan(\theta) = (X_L - X_C)/R$

2. RLC circuits

3. Power = energy/time = I V; $P_{avg} = I_{rms} V_{rms} \cos(\theta) = I_{rms}^2 R$

4. resonance: inhomogeneous differential equation predicts

   maximum I (and hence maximum P) when $\omega_{applied} = \omega_{natural}$

Supplementary Problems (S-):

37) Consider a series RLC circuit with an oscillating voltage source of 110 volts (rms) at 60 Hz. The resistance is 86 Ω, the capacitance is 8 μF, and the inductance is 70 mH.
a) What is the capacitive reactance (X_C) ?
b) What is the inductive reactance (X_L) ?
c) What is the impedance (Z) of the circuit?
d) What is the rms current throught this circuit?
e) Explain why $V_{ac}(t) = V_L(t) + V_C(t) + V_R(t)$ but $V_{AC\text{-}rms} \neq V_{L\text{-}rms} + V_{C\text{-}rms} + V_{R\text{-}rms}$.

38) a) What is the resonance frequency of the RLC circuit in the above problem? b) What is the impedance of the circuit at the resonance frequency? c) What is the rms current through the circuit if the voltage remains at 110 volts rms but the frequency is at resonance?

Answers to Supplementary Problems:

37) (a) 331.6 Ω; (b) 26.4 Ω; (c) 317.1 Ω; (d) .35 amps-rms; (e) you are on your own!
HINT: consider conservation of energy, definition of voltage, and phase angles.

38) (a) 213 Hz; (b) 86 Ω; (c) 1.28 Amps-rms.

Magnetism in Matter

A. Magnetization   S-39

1. M (due to material) Magnetization in Amp-turns/meter

2. H (due to external currents) Magnetic Field Strength or Intensity in Amp-turns/meter

3. M, B, H: $B = \mu_o (H + M)$; $B = \mu H$; $\mu = \mu_o (1 + X)$; $M = X H$
   where B is Magnetic Flux Density or Magnetic Field;
   μ is magnetic permeability, X is magnetic susceptibility

B. Types

1. Paramagnetism: $\chi \ll 1$; $\chi > 0$

2. Diamagnetism: $|\chi| \ll 1$; $\chi < 0$

3. Ferromagnetism: $\chi \gg 1$

C. Hysteresis: $B = \mu H$; but μ not really constant: depends on past history

Supplementary Problem (S-):

39) Consider a (long) solenoid with 2000 turns wrapped around a cylinder of radius 4 cm and length .57 meters that carries a current of 2 amps. (a) If air is inside the solenoid, what is the magnitude of the magnetic field (B) in the solenoid? (b) What is the magnitude of the magnetic field intensity (H) in the solenoid? (c) If the cylinder is filled with iron (assume $\chi_M = 5000$) instead of air, what is the magnitude of the magnetic field (B) in the solenoid? (d) Does the magnitude of the magnetic field intensity (H) increase, stay the same, or decrease when the iron is put in place of the vacuum?

Answers to Supplementary Problems:

39) a) 8.82×10^{-3} T = 88.2 G; b) 7017 Amp-turns/m; c) 44.11 T; d) stay the same.