Study Guide for Part Three
MAGNETIC FIELDS

Magnetic Fields

A. Review: forces and fields
   1. gravitational force: $F_g = G\, m_1\, m_2 / r^2$, $\mathbf{g} = \mathbf{F}_g/m$
   2. electric force: $F_e = k\, q_1\, q_2 / r^2$, $\mathbf{E} = \mathbf{F}_e/q$
   3. magnetic force: $F_m = X\, p_1\, p_2 / r^2$, but no monopoles!

B. Magnetic Force - on moving charges!
   1. definition: $\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$ (define **B** this way)
   2. Units: Tesla = Weber/m² = 10^4 Gauss
   3. $\times$ is cross product: magnitude: $F = q\, v\, B \sin(\theta_{vB})$
      direction: right hand rule

C. Motion of charge in magnetic field
   1. v perpendicular to B (so $F \perp v$, so $a \perp v$, so circular motion) S-23
   2. v parallel to B (so $F_m = 0$, so $a = 0$, so $v =$ constant)
   3. v in general

D. Applications I: S-24,25
   1. mass spectrometer note: 1 amu = 1.66×10^{-27} kg
   2. cyclotron & syncrotron
   3. magnetic bottle
   4. Hall effect

E. Current consists of moving charges: $v\, dq = I\, ds$ S-26
   1. $F = \int I\, d\mathbf{s} \times \mathbf{B}$
   2. torque on a current loop [recall: $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = r\, F \sin(\theta_{rF})$, direction: right hand rule]

F. Applications II: S-27
   1. electric meters
   2. electric motors

Supplementary Problems (S-):

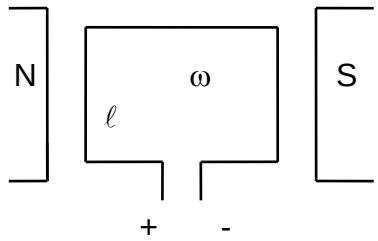
23) An electron is moving East with an initial speed of 3×10^4 m/s through a magnetic field directed up of strength 52 Gauss. a) What is the magnitude and direction of the magnetic force on the electron? b) What is the magnitude of the acceleration of the electron?

24) A certain mass spectrometer has singly ionized atoms with speed 4×10^5 m/s entering a magnetic field of strength 0.8 T. If the ions have a diameter of 13.5 cm in the field, what is the mass of the ions in kilograms? in amu? Will C^{12} ions have a bigger or smaller radius?

25) A cyclotron consists of a pair of "dees" with radius 8 cm placed in a constant and uniform magnetic field of strength 1.2 Teslas. An accelerating voltage of 1000 volts is employed. If the cyclotron is used to accelerate alpha particles (q=+2e, m=4 amu), a) what will the maximum speed of the alpha particles be from this cyclotron? b) What will the energy of the alphas be when they emerge with this maximum speed (answer in Joules and eV's)? c) What should the frequency of alternating the accelerating voltage be? d) Will the maximum energy of the alphas

be increased by (1) increasing the accelerating voltage? (2) by increasing the radius of the "dees"? (3) by increasing the magnetic field strength?

26) A square loop of wire of width 6.3 cm and length 4.5 cm is placed in a region where the magnetic field is .0074 Teslas in strength as shown in the figure below. A current of 3.7 Amps flows through the wire. (a) What will be the magnetic force (magnitude and direction) on the left side of the wire? (b) What will be the magnetic force (magnitude and direction) on the top side of the wire? (c) Will the force on the left side be equal and opposite to that on the right side (yes or no)? (d) Which direction will the total magnetic force on the square current loop be? (specify direction or answer none if force is zero) (e) Which direction will the total magnetic torque on the square current loop be? (specify direction or answer none if torque is zero)



27) An electric motor has 1000 windings wound around an area of 45 cm² in a magnetic field of 0.3 T. If a current of 5 amps flows through the windings, what is the (a) maximum torque produced? (b) average torque?

Answers to Letter Problems:

23) a) 2.5×10^{-17} Nt directed North; b) 2.75×10^{13} m/s².

24) 2.16×10^{-26} kg; 13 amu; C^{12} will have a smaller radius than C^{13};

25) a) 4.63×10^{6} m/s; b) 7.11×10^{-14} Joules = 444 KeV; c) 9.2 MHz;

d) (1) No (only the number of cycles of the alphas will be decreased), (2) Yes, (3) Yes.

26) a) 1.23×10^{-3} Nt, down; b) zero; c) Yes; d) none; e) toward the bottom of the page.

27) a) 6.75 Nt m; b)4.3 Nt m.

Sources of Magnetic Fields

A. **Biot-Savart Law**
1. gravitational field & mass: $\mathbf{F} = m\,\mathbf{g}$, $\mathbf{g} = G\,M/r^2$ directed toward M
2. electric field & charge: $\mathbf{F} = q\,\mathbf{E}$, $\mathbf{E} = k\,q/r^2$ directed away from +, toward -
3. magnetic field & moving charge:
$\mathbf{F} = (I\,d\mathbf{s}) \times \mathbf{B}$, $\mathbf{B} = X(I\,d\mathbf{s}) \times r/r^2$ where $X = \mu_o/4\pi$ and r is a unit vector

B. Special cases: S-28
1. B due to straight wire $= \mu_o I [\cos(\theta_2) - \cos(\backslash\theta_1)] / (4\,\pi\,r)$
2. B at center of loop $= \mu_o I / (2\,r)$
3. B on axis of loop $= \mu_o I R^2 / (2\,r^3)$
4. B for solenoid $= \mu_o n I [\sin(\theta_2) - \sin(\theta_1)] / 2$, where n = N/L

C. **Ampere's law:** $\mu_o I = \oint \mathbf{B} \bullet d\mathbf{s}$ S-29

1. B due to LONG straight wire $= \mu_o I / (2 \pi r)$
2. B due to LONG solenoid $= \mu_o n I$, where $n = N/L$

D. **Gauss's Law for Magnetism** and Magnetic Flux: $\oiint \mathbf{B} \times d\mathbf{A} = 0$

(no magnetic monopoles)

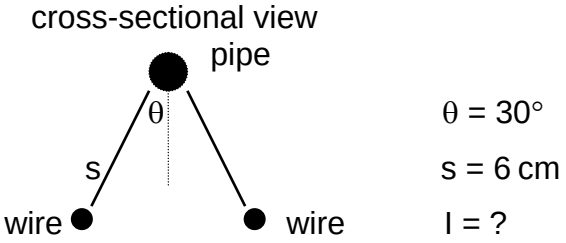
E. Force between currents S-30

Supplementary Problems (S-):

28) A long wire runs carries a current of 300 amps from the North to the South.
a) What is the magnetic field due to the current at a location 2 cm East of the wire?
b) What is the magnetic field due to the current at a location 12 meters below the wire?
c) What is the acceleration of an electron moving South at a speed of 2×10^6 m/s at the location 12 meters below the wire if the magnetic force is the only force acting on an electron?

29) a) What is the magnitude of the magnetic field at the center of a solenoid of length 75 cm that has 300 turns wrapped around a cylinder of radius 7 cm and that has a current of 2 amps flowing through the wire?
b) How much difference is there between the long solenoid approximation and the exact result?

30) [Extra Credit (+4): Two wires are parallel to each other and parallel to a pipe from which they hang. The wires are very long and are a constant 6 cm from the pipe. (See figure below.) When a certain current is run through the wires, they separate from each other and each makes an angle of 30° from the vertical. Each wire has a mass of 0.13 grams per meter. Is the current in the two wires in the same direction or opposite? What is the current through each?]



Answers to Letter Problems:

28) (a) 3×10^{-3} T = 30 Gauss directed up; (b) 5×10^{-6} T = .05 Gauss directed East;
(c) 1.76×10^{12} m/s² directed down away from the wire.

29) (a) 1.001×10^{-3} T = 10.01 Gauss; (b) approx = 10.05 Gauss, so difference is only 0.4% .

30) opposite direction; 14.85 amps.