Study Guide for Part One

ELECTRIC FIELDS

Electric Fields

A. Coulomb's law: basic electric force

1. new force: $F_e = k\, q_1\, q_2 / r^2$
2. similar to gravity: $F_{gr} = G\, m_1\, m_2 / r^2$ S-1
3. direction: can be either attractive or repulsive
4. constant: $k = 1/(4\pi\varepsilon_o) = 9 \times 10^9 \text{Nt m}^2/\text{Coul}^2$ (like $G = 6.67 \times 10^{-11} \text{Nt m}^2/\text{kg}^2$)
5. recall: force is a vector! (work in components) S-2

B. Electric field

1. definition: $\mathbf{E} \equiv \mathbf{F}(\text{on } q_{test})/q_{test}$
2. for a point charge: magnitude: $E = k\, q / r^2$;
   direction: away from + charge; toward - charge.
3. for several point charges: add as vectors! S-3,4

$$E_x = \Sigma\, kq_i \cos(\theta_i)/r_i^2 \; ; \qquad E_y = \Sigma\, kq_i \sin(\theta_i)/r_i^2$$

4. for an extended charged object: add or integrate:

$$E_x = \int k\, dq \cos(\theta)/r^2 \; ; \qquad E_y = \int k\, dq \sin(\theta)/r^2$$

a) line ($dq = \lambda\, dx$) S-5

b) on axis of a ring ($dq = \lambda\, d\ell$)

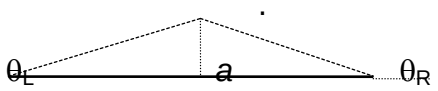
Supplementary Problems (S-):

1) At what point between the earth and moon will an object neither fall toward the earth or toward the moon? (Neglect the motions of the earth and the moon, and neglect the gravity of the sun.) The earth-moon distance is 384,000 km; the radius of the earth is 6,400 km; the radius of the moon is 1,750 km; the mass of the earth is 6.0×10^{24}kg; the mass of the moon is 1.23% that of the earth.

2) At a particular instant a particle of mass $m_A = 5$ mg and charge of $q_A = 5$ μCoul is located at the origin. A second particle of mass $m_b = 2$ mg and charge of $q_B = -6$ μCoul is located at that same instant at a position (-5 m, +3 m) relative to the origin.
a) What is the force on q_A due to the presence of q_B ?
b) Is the magnitude of the force on q_B due to the presence of q_A the same, greater or smaller than the magnitude of the force on q_A due to the presence of q_B ?
c) Assuming the two charges have no other forces on them, what is the magnitude of the resulting acceleration of particle A due to the electrical interaction with particle A?
d) Is the acceleration of particle B due to the presence of particle A the same, greater or smaller than the magnitude of the acceleration of particle A?

3) Consider two charges: q_A has a charge of 5 nCoul and is located at (5 m, 3 m); q_B has a charge of -4 nCoul and is located at (7 m, -2 m).
a) What is the electric field at the origin due to q_A alone?
b) What is the electric field at the origin due to BOTH charges?

4) A horizontal wire of length ℓ has a total charge Q = 2.0 μC uniformly distributed on the wire. Calculate the x and y components of the electric field at a field point located a distance *a* above the center of the wire. Do this by breaking the wire into five pieces; then finding the charge of each piece; then, assuming that this charge is a point charge located at the center of each piece, finding the x and y components of the electric field at the field point from each piece; and then adding up the fields due to each of the five pieces to get the total x and y components of the electric field.



a = 5 cm ℓ = 20 cm Q = 2.0 μCoul

5) For the charged wire of problem 4 above:
a) What is the charge density, λ, on the wire?
b) What is the angle θ_L that the line from the left end of the wire to the field point located a distance *a* above the center of the wire makes with the wire?
c) What is the electric field strength E at this field point?
d) What is the direction of the electric field at this field point?
e) Compare your answer to the approximate answer from the numerical method of problem 4 above.

Answers to Supplementary Problems:

1) 345,660 km from earth.

2) (a) 7.94×10^{-3} Nt; (b) same; (c) 1.59 m/s²; (d) acceleration of B is larger.

3) (a) E = 1.32 Nt/Coul at 211°; (b) E = .99 Nt/Coul at 241°

4) $E_x = 0$ Nt/Coul; $E_y = 3.240 \times 10^6$ Nt/Coul

5) (a) 10.0 μC/m; (b) 26.2°; (c) 3.220×10^6 Nt/Coul; (d) straight up (away from the wire).

Gauss' Law

A. Flux – definition

1. example: volume flow and velocity vector: $dV_{ol}/dt = \oiint \mathbf{v} \bullet d\mathbf{A}$

2. electric flux = $\oiint \mathbf{E} \bullet d\mathbf{A}$

B. Gauss' law: $\oiint \mathbf{E} \bullet d\mathbf{A} = 4\pi k Q_{enclosed} = Q_{enclosed}/\varepsilon_o$

1. statement of the law

2. using the law to find E (need symmetry to do integral)

   a) hollow sphere S-6

   b) solid sphere

   c) cylinder S-7

   d) plane S-8

Supplementary Problems (S-):

6) A hollow conducting sphere of radius 12 cm has a charge of 7 nCoul placed on it. Assume there are no other charges in the area.
a) What is the magnitude and direction of the electric field 1 cm below the top of the sphere (inside the sphere) ?
b) What is the magnitude and direction of the electric field 1 cm above the top of the sphere (outside the sphere) ?

7) A coaxial cable (assume a long cable) has an inner wire of radius 2 mm and an outer cylinder of radius 3 cm. A charge density of -40 nCoul/m is placed on the inner wire and a charge density of +50 nCoul/m is placed on the outer cylinder. Assume vacuum between the cylinder and the wire.
a) What is the electric field 1 cm from the axis (center) of the wire (8 mm from the surface of the wire, between the wire and the cylinder) ?
b) What is the electric field at a radius of 4 cm (1 cm from the surface of the cylinder outside the cable) ?

8) Two plates are separated by 2 cm and are 200 cm by 200 cm in size.
a) If a charge of +5 μCoul is transferred from an originally neutral bottom plate to a top plate (also originally neutral) which is parallel to the bottom plate, what is the electric field 0.5 cm below the top plate directly underneath the center of the top plate? (Assume a uniform distribution of the ±5 μCoul on the top/bottom plate.)
b) Does the distance from the top plate to the field point affect the value of the field?
c) If an electron were placed at the point 0.5 cm below the top plate, what would be the force on the electron?
d) What would the acceleration of the electron be?
e) Would this acceleration increase, stay the same, or decrease as the electron moved from this point in the direction of the acceleration?
f) If the electron started at the negative plate from rest, how fast would it be going when it reached the positive plate?

Answers to Supplementary Problems:

6) (a) 0 Nt/Coul (no direction);

(b) 3.73×10^{3} Nt/Coul directed up.

7) (a) 7.2×10^{4} Nt/Coul directed toward the axis;

(b) 4.5×10^{3} Nt/Coul directed away from the axis.

8) (a) 1.4×10^{5} Nt/Coul directed from top to bottom plate;

(b) the 0.5 cm distance does not affect the field unless it puts it outside the plates instead of between the plates;

(c) 2.24×10^{-14} Nt directed from the bottom plate to the top plate;

(d) 2.46×10^{16} m/s² directed from the bottom plate to the top plate;

(e) stay the same;

(f) 3.14×10^{7} m/s.