THIN LENSES: BASICS

OBJECTIVE: To study and verify some of the laws of optics applicable to thin lenses by determining the focal lengths of three such lenses (two convex, one concave) by several methods.

THEORY: Each point of a self-luminous object or reflection object (O) is a source of light with a large number of rays emanating from it in all directions. A lens will alter the direction of those rays which strike it, and an image (I) of the object may be formed. If rays from a single point on the object strike different parts of the lens and eventually all intersect at some other point in space, a ***real*** image is formed. Real image formation is illustrated in Fig. 1. A ***virtual*** image is formed if the **projections** of the rays from a single point on the object intersect at a single point in space. A screen placed at this point will not reveal an image, but the image is visible to the brain if observed from the proper position such that the diverging rays enter the eye. See Fig 2.

A ***thin lens*** is one whose thickness is negligible in comparison to the image and object distances [*object distance* is measured from object to lens; *image distance* is measured from lens to image position]. A ***convex*** lens is thicker in the center than at the edges. Such a lens is also called a *positive* lens or a *converging* lens. A ***concave*** lens is thinner at the center than at the edges, and is also called a *negative* lens or a *diverging* lens.

The axis of a lens is the line drawn through the centers of curvature of its refracting surfaces. If a beam of rays parallel to the axis is incident on a converging lens, the beam is brought to a focus at a point on the axis called the *focal point* (*F*). If a beam of rays parallel to the axis is incident on a diverging lens, the rays diverge as though radiated from a point on the axis. This point is also called the *focal point* (*F*). The distance along the axis from the focal point to the center of the lens is known as the ***focal length*** (*f*). A converging lens has a positive focal length; a diverging lens has a negative focal length.

There are at least three commonly used symbols for object and image distances:

object distance:	s (used here) ,	o (sometimes used) ,	p (sometimes used)
image distance:	s' (used here) ,	i (sometimes used) ,	q (sometimes used)

These distances and the focal length are related by the "thin lens equation",

$$1/s + 1/s' = 1/f . \qquad (1)$$

Object distances, image distances, and focal lengths can be positive or negative. We will see examples of both positive and negative values in this experiment.

The ratio of the image size (h') to the object size (h) can be predicted by use of the *lateral* ***magnification*** *factor*

$$M_{exp} \equiv h' / h . \qquad (2)$$

[NOTE: If the image is inverted relative to the object, then h' is negative, making M negative.]

As seen in Fig. 1, geometry gives a theoretical lateral magnification in terms of the object and image distances (from h/s = -h'/s'):

$$M_{the} = -s' / s \,. \qquad (3)$$

[NOTE: The minus sign in Eq. (3) is necessary to keep the signs of the experimental and theoretical magnifications consistent. That is, when *s'* is positive, *h'* is negative.]

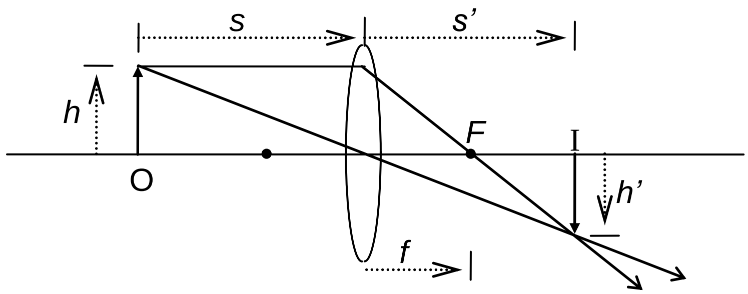
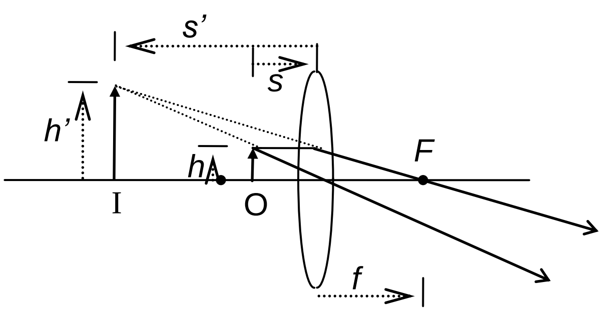
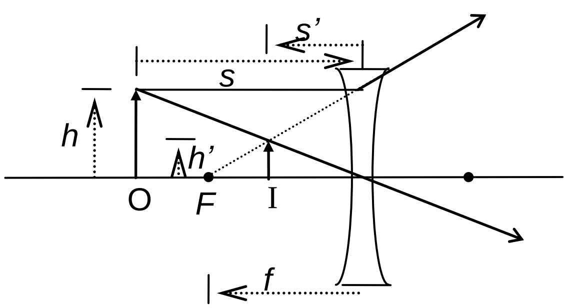


Fig. 1: Real Image Formation

Fig. 2: Virtual Image Formation

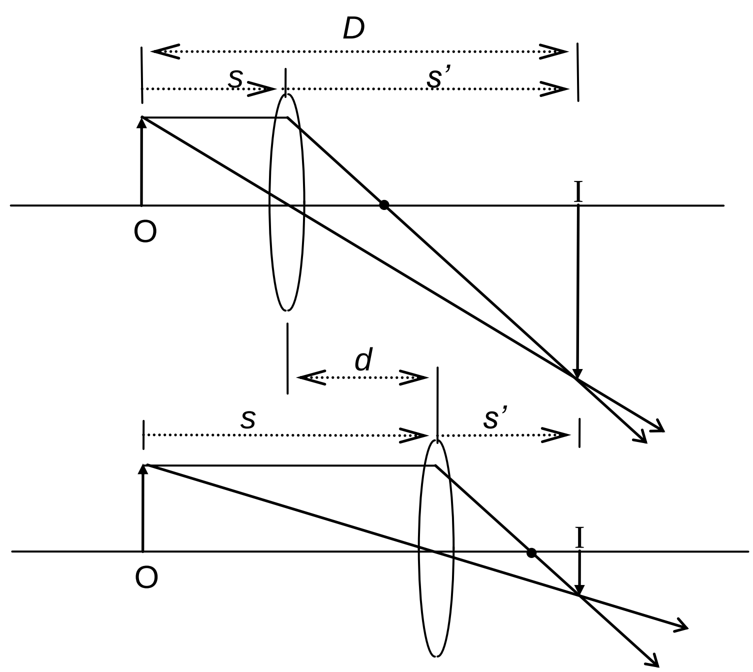


(a) with converging lens



(b) with diverging lens

Fig. 3: Conjugate Foci



Part 1: Convex Lens

Three methods will be used to measure the focal length of a convex lens:

1. **Parallel rays:** If the object is sufficiently far away so that *s>>f*, the incoming rays will be almost parallel, and the object will be focused at approximately the focal point of the lens. [For s large, $1/s \approx 0$, so $0 + 1/s' = 1/f$, so $s' \approx f$.] This method is used in Step 1.

2. **The thin lens equation:** The image and object distances will be measured and *f* calculated with Eq. (1) . This method is used in Step 2. Since object and image distances will be measured, we will also investigate lateral magnification in this step.

3. **Conjugate foci:** The locations of the object and image on the axis of a lens are called the conjugate foci. For a given object distance, s_1, an image is formed at a distance, s_1', determined by the thin lens equation. If the object is moved to the image location, $s_2=s_1'$, a new image will be formed at the original object position, $s_2'=s_1$. Thus, a lens has a symmetry with respect to its conjugate foci (*s* and *s′* can be interchanged in Eq. (1)). In the lab the position of the lens will be changed to effect the interchange of object and image while keeping the positions of the object and the screen unchanged. Referring to Fig. 3, if $D = s + s'$ and *d* equals the shift in position of the lens, $d = s_1 - s_2 = s_1 - s_1'$, then one can show that

$$f = (D^2 - d^2) / 4D \, . \qquad (4)$$

   This method is used in Steps 3 and 4.

PROCEDURE: The nominal focal lengths on the lenses are in mm, but the focal lengths below are in cm so they are more easily compared to the object and image distances usually measured in cm.

1. See how each of the three lenses (+5 cm convex, +10 cm convex, and –15 cm concave) acts when used as a **magnifying glass**. Do all the lenses magnify? Which is the "strongest" lens, i.e. which magnifies the most when used as a magnifying glass?

2. Place only the +10 cm convex lens and the white screen on the optical bench and aim the lens at some object outside the room. (If possible, perform this part of the experiment in the hallway with the hallway lights off and with the lights in a far room on.) Move either the lens or the screen along the bench until the image is focused as sharply as possible. If an image can be focused, **measure and record** the image distance, which in this case is approximately the same as the focal length. Can you justify this assumption? [HINT: Consider the thin lens equation with s very large so that $1/s \approx 0$.]
   Replace the +10 cm convex lens with the +5 cm lens and repeat the above procedure.
   Replace the convex lens with the concave lens and repeat the above procedure.

3. FOR PARTS 3, 4 & 5, USE ONLY THE WEAKER CONVEX LENS.
   a) FOCAL LENGTH. Place the light box with slide on the optical bench. By shining a light through the slide, we will use the +10 cm convex lens to form an image of the slide on the screen. Hence the slide will be our object in this part. Next place the lens at some position beyond the focal length, i.e. s>f, and focus an image of the object on the screen by moving the screen. **Measure** object and image distances, *s* & *s′*, and **calculate** the focal length, *f*, using Eq. (1).

b) MAGNIFCATION. The length of one of the object arrows is h = 30 mm, and the circles on the object have diameters of 10 mm and 20 mm. **Measure** the length of the image arrow, h'. (If you cannot image the entire arrow clearly, then use the large or small circle as your image and measure the diameter of the image circle.) This length of the image is the value of h'. Use the appropriate value for h. Note whether the image is inverted and use the proper sign for h'. **Calculate** the **experimental** and **theoretical** magnifications using Eqs. **(2)** & **(3)** and compare the values.

Repeat these measurements (for s, s', and h') and calculations (for f, M_{th}, and M_{exp}) for at least **two more** object distances. Be sure to have at least one case where the image is magnified (bigger than object) and at least one case where the image is demagnified (smaller than object). **Average** all of your focal length values to get a good value for the focal length.

4. Referring to Fig. 3, if the image and object distances are equal ($s = s'$), then $D = 2s$, $d = 0$, and Eq. **(4)** gives $f = s/2$. (The same result is found from the thin lens equation.) This would put the image a distance of $4f$ from the object. See if this is true experimentally by doing the following: Using the average value of f from Step 3, place the lens a distance of $2f$ from the object and move the screen until the image is formed. **Measure** the distance between object and image. Is it $4f$? **Record** the difference between $4f$ and the measured object to image distance.

5. Place the screen at a distance from the object that is greater than $4f$. This is the distance D. By moving the lens but not the object or screen, find the **two** lens positions that give sharp images. **Measure** the distance between these two lens positions. This is the distance d. Using D and d in **Eq. (4)**, **compute** the focal length and **compare** with the average value of f obtained in part 3.

REPORT:

1. For each of the procedures above:
   1a) draw a diagram,
   1b) record your data,
   1c) show your calculations,
   1d) state your results, and
   1e) answer any questions posed.

   In each case, be sure to include a discussion of possible errors and the estimated accuracy of your data and how this uncertainty in data affects your results.

2. Compare the values of the focal length obtained from the above procedures. Comment on whether they agree with one another within experimental uncertainty.

3. Comment on your values for magnification. Is the **theoretical value** the same as the **experimental value** within experimental error? Is there just one value for magnification for this lens? If not, what does the magnification depend upon?

Part 2: Concave Lens

The method of determining the focal length of a **concave** (negative, diverging) lens will involve using it in conjunction with an auxiliary convex (positive, converging) lens. In Fig. 4, I_1 is the image of the object O_1 which would be formed by lens #1 if the diverging lens #2 where not present. But with lens #2 in place, the rays do not converge as quickly, and the image I_2 is formed farther away. When analyzing multiple lens systems, the image formed by the first lens is considered to be the object of the second lens ($I_1 \rightarrow O_2$), and the thin lens equation is used.

PROCEDURE:

1. IMAGE AS OBJECT.
   (a) Before we consider a concave lens with a convex lens, let's work with ***two convex*** lenses to see more clearly how to work with two lenses. Start with the weaker of the two convex lenses and focus an image on the screen. Try to get an image that is somewhat close in size to the object. **Record** the object distance, image distance, object height, and image height. **Calculate** the focal length of this weaker lens. Compare to the values you have already obtained for it in Part 1.
   (b) Next note the position of the screen, remove the screen (or place it further away) and place the stronger convex lens about 10 cm beyond the original position of the screen (where the image of the first lens is). The distance from the original screen position to the second lens (about 10 cm) will be the object distance, s_2, for the second lens. Now relocate the screen so that it is beyond the second lens and see if you can focus an image. If you can, **record** the new position of the screen and calculate the image distance, s_2', for this second lens, and record the final image height, h_2. Now **calculate** the focal length of this second (stronger) lens. Should the stronger lens have a focal length bigger or smaller than the weaker lens? Does it? Is this focal length close to what you determined in Part 1, procedure 1 (hallway method) for this stronger lens?

2. FOCAL LENGTH OF CONCAVE LENS.
   (a) Remove the second stronger convex lens and re-form an image with the weaker **convex** lens, just as you did above. **Record** the position of the image since this will become the object for the concave lens. You should have the same screen location as in part 1a above. Without moving either the object or convex lens, place the **concave** lens on the bench BETWEEN the convex lens and the screen. (NOTE: We cannot position the lens behind the initial image, as we did in 1a above, since the light will be diverging after it forms an image, and a concave lens does not converge the light diverging from the image the way a convex lens does. Instead, we will use the concave lens to diverge the already converging light coming from the convex lens **before** it forms an image. If the concave lens is weaker than the convex lens, the rays will still form an image but further away as shown in Fig. 4.)
   (b) Now reposition the screen until a new image is formed. **Record** the position of the concave lens and the position of the new screen (and image). The distance between the 2^{nd} lens and the second screen is our new image distance, s_2'. Since the object for the concave lens (image from the convex lens) is on the "wrong" side of the lens and the light does not focus a real image at the position, we have a **virtual** object and our object distance, s_2, for the concave lens will be **negative**. Now **compute** the focal length, f, of the concave lens. A concave lens should have a negative focal length. Does yours?
   (c) **Repeat** the above procedure in 2b by moving the concave lens a couple cm and refocusing the image (adjusting the 2^{nd} screen position). This will change both s_2 and s_2'. **Recalculate** f. Compare your two values for the focal length of the concave lens. Are they the same? Should they be?

3. MAGNIFICATION.
   (a) **Measure** the height of the final image on the screen, h_2'. Using h'_1 (image size of convex lens) as the object size for the concave lens, h_2, **calculate** the magnification both **experimentally** (h_2'/h_2) and **theoretically** (s_2'/s_2) for the concave lens. Are the object and image both upside down? Should $M_{concave}$ be positive or negative?
   (b) Now **determine** the experimental magnification of the **system** of lenses. This is the final image size (h'_2) divided by original object size (h_1). Does $M_{final} = M_{convex} + M_{concave}$ or does $M_{final} = M_{convex} * M_{concave}$?

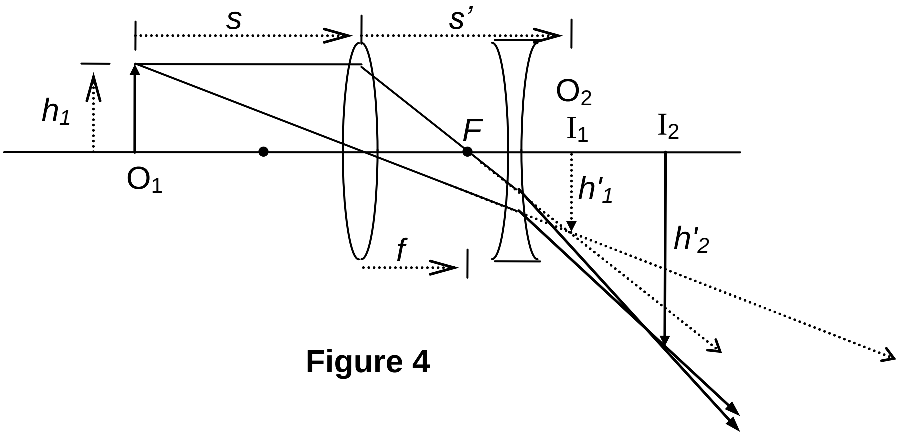


Figure 4

REPORT:

1. For each of the steps above:
   1a) record your data,
   1b) show your calculations,
   1c) state your results, and
   1d) answer any questions posed.

   In each case, be sure to include a discussion of possible experimental errors (uncertainties) and the estimated accuracy of your data and how this uncertainty in data affects your results.

2. Show from theory that $M_{final} = M_{convex} * M_{concave}$. In fact, for a system with N lenses, the final system magnification is simply the product of all of the individual magnifications,

$$M_{final} = M_1 * M_2 ... * M_N = \prod_{j=1}^{N} M_j \ .$$