

THE VIBRATING STRING

OBJECTIVE: To observe waves on a slinky, to experimentally determine if and how waves reflect, and to see the phenomenon of resonance as it applies to waves on a string.

THEORY: When a force sets up a disturbance (y) on a string, Newton's Second Law of motion ($\Sigma \vec{F} = m\vec{a}$) can be used to get a wave equation which predicts that the disturbance will propagate down the string with a velocity (v) that depends on the tension in the string (T_{ension}) and the linear mass density of the string (μ) according to:

$$v = [T_{ension}/\mu]^{1/2} \quad (1)$$

where the linear mass density, μ , is defined as

$$\mu = M_{total} / L_{total} . \quad (2)$$

This speed of the disturbance on a string is too fast to allow one to see the actual motion of the disturbance along the string. However, we can see the propagation of a disturbance (or pulse) on a slinky since the speed is much lower. We will observe this pulse propagation in Part One. We refer to a disturbance or pulse that travels as a *wave*.

If the pulse hits an end of the string (or slinky) that is tied down, then the pulse does **reflect** from that tied down end in such a way that the incident wave and the reflected wave add together at the tied down point so as to give a zero displacement at the end. What this means, in effect, is that the wave undergoes a reversal of amplitude, i.e. it “flips” or becomes inverted. We will see this in Part Two for a wave on a slinky.

If one end of a string is held fixed and the other end is attached to a vibrator so its direction of vibration is at right angles to the direction of the string, wave disturbances will continuously travel along the string with the velocity v of Eq. (1) and start **interfering** (adding) with previous parts of the wave that have reflected back from the tied down end. Since the other end is also constrained by the vibrator, then waves will reflect from that end as well causing many reflected waves to interfere with the incident wave. Normally, this interference between incident and the many reflected waves will cause the resulting wave to have about as many up (or positive) contributions as down (or negative) contributions causing approximately zero resultant amplitude. However, when the cause of the disturbance is periodic (sinusoidal) and when the length of the string (from vibrator to pulley), $L_{vibrating}$, is at (or very close to) an integer number, n , of half-wavelengths ($\lambda = \text{wavelength}$),

$$L_{vibrating} = n(\lambda / 2) , \quad (3)$$

then we get **constructive interference** at certain points along the string resulting in what is called a *standing wave* pattern. Also, when the waves are sinusoidal (or *harmonic*), the wavelength is related to the frequency in a simple way:

$$\lambda f = v . \quad (4)$$

This relation comes from the basic definition of speed as distance (here λ) divided by time (here the period which is $1/f$).

In the Part Three of the experiment we will employ a sinusoidal oscillator with a set frequency of 120 Hz. We can measure both the mass and length of a string and use **Eq. (2)** to get μ . We will control (and measure) the tension in the string (T_{ension}) by using weights hung over a pulley. Then by observing when we get **standing waves**, we can use **Eq. (1)** to determine the phase velocity, v , of the wave and hence determine the wavelength, λ , of the waves on the string from **Eq. (4)**. We can then **test** to see if **Eq. (3)** holds true for standing waves.

Part 1: Wave Propagation

PROCEDURE:

1. Stretch the slinky to some distance and try to measure the speed of a pulse that you put on the slinky by flicking your hand up and down. Remember that speed is simply the distance traveled divided by the time it takes to go that distance. Try to estimate the speed of the pulse as best you can.
2. Stretch the slinky to some other distance and again try to measure the speed of the pulse that you put on the slinky. Did the speed change when you changed the conditions on the slinky?
3. In Steps 1-2, you were examining a **transverse** wave. The medium supporting the pulse (the slinky) moved up and down, perpendicular or transverse to the direction that the pulse traveled. A wave that travels in a medium where the medium moves back and forth in the same direction that the wave travels is called a **longitudinal** wave. You can send a longitudinal wave along a slinky by stretching the slinky while it is lying on a desk or the floor and rapidly pushing and pulling on one end. Produce such a longitudinal wave on the slinky and observe its motion.

REPORT:

1. Record your estimates of the speed of the pulse in Steps 1-2. What did stretching the slinky do to the speed of the wave?
2. Is light a transverse or longitudinal wave? What about a sound wave? What about a wave on a string?

Part 2: Reflection of Waves

PROCEDURE:

1. Stretch the slinky to some distance and put a pulse on the slinky so that the pulse is up. Then watch to see if the pulse reflects (bounces) off of a fixed end (your partner?). If it does reflect, notice whether the pulse stays up or becomes inverted (pulse is down).
2. Now stand on something high (the desk?) and hold the slinky up so that the other end does NOT touch the ground (the end is free). Now put a pulse on the slinky (pulse is away from you). Notice whether there is a reflection in this case, and if so whether the pulse stays away from you or becomes inverted (points toward you) on reflection.

REPORT: Record your observations.

Part 3: Interference and Standing Waves On a String

PROCEDURE:

- Note that the frequency, f , of the vibrator is 120 cycles/sec or **120 Hz**. This is determined by the construction of the vibrator and that the voltage (and current) from an outlet in the U.S. alternates 60 times a second.
- Measure a length** of the same type of string that we will vibrate, L_{total} , with a meter stick, and **weigh** it on a balance to get M_{total} . (The string should be at least 2 meters long. Weigh it with another similar string if your string is less than 2 meters long.) Use **Eq. 2** to **calculate** the linear density (μ) of this type of string.
- Connect one end of the string to the vibrator and the other end to a weight hanger hung over a pulley. **Measure the length** of the string from vibrator to pulley. This is a different length from what you measured in step 2 above. This length is the $L_{\text{vibrating}}$ used in **Eq. 3**. It should be at least 1.5 meters.
- Start the vibrator, and increase the tension (T_{ension}) in the string by adding weights [$T_{\text{ension}} = m_{\text{weights}}g$] until there are several loops in the string. When in proper adjustment the amplitude of vibration will be a maximum. Even though points on the string are moving up and down, it appears as if the string is 'standing' in mid-air with several loops along its length. The place where the **maxima** occur are called **anti-nodes**; the places where the string **does not** vibrate are called **nodes**. The length of each loop should equal a half-wavelength. Because the number of loops must fit between the end points of the string, the condition for a **standing wave** is given by **Eq. (3)**, $L_{\text{vibrating}} = n(\lambda / 2)$, where n is the number of loops. (If you think about it, n must also be the number of anti-nodes and the number of nodes *plus one*.) **Record** the hanging mass, m_{weight} , and the number of anti-nodes, $n_{\text{experiment}}$. Note that the tension in the string is equal to the weight ($m_{\text{weight}}g$) of the hanging mass, m_{weight} . Why?
- Repeat the above procedure, each time increasing the tension so as to reduce the number of loops, until **at least three sets** of measurements have been obtained. One of the sets should have a number of loops of 4 or less.

REPORT:

- For each standing wave, **calculate** the speed of the wave, v , using **Eq. (1)**.
 - Then **calculate** the wavelength, λ , using **Eq. (4)**.
 - Now using the measured length of the string, $L_{\text{vibrating}}$, and the wavelength calculated in step b, λ , use **Eq. (3)** to **calculate** the predicted number of loops, n_{theory} . (Recall that n equals the number of loops or the number of anti-nodes.)
 - Compare** the predicted number of loops, n_{theory} to the observed number of loops, $n_{\text{experiment}}$.
 - Consider possible **sources of uncertainty** and see if these are sufficient to explain the percent difference. To do this, consider your possible sources of uncertainty as percentages.
- What does an increase in tension do to the following quantities: speed of wave, wavelength, number of nodes ?

3. What tension would be required to produce vibration in the fundamental mode ($n = 1$) in the string used in this experiment. Would it be reasonable for you to get this fundamental mode?

An Application:

For a guitar, there is no pre-determined frequency that vibrates the strings. Instead as each string is plucked, that string will oscillate at one or more of the resonant wavelengths determined by the above equations using the tensions, mass densities, and vibrating lengths of the guitar's strings.

For a guitar, the velocity of the wave on a guitar string is determined by the linear mass density of the string (notice on a guitar that different strings have different thicknesses) and by the tension in the string (notice that to tune a guitar you must adjust the tension in the strings by knobs at the top of the neck of the guitar). See **Eq. (1)**. When you play a certain note on a guitar string, you must press the guitar string at a certain position. This position then determines the vibrating length of the string, and hence the possible wavelengths that will sound (resonate). Then from **Eqs. (3) & (4)**, there will be only certain frequencies (based on the integers, n) supported on the string, and these certain frequencies combine to form that certain note on the guitar. If a string is plucked, the lowest such frequency (longest wavelength where $n=1$) called the **fundamental frequency** is heard along with some higher frequencies (where $n>1$), called harmonics. The specific harmonics are based on where the string is plucked and the shape of the guitar. It is these higher harmonics that differentiate the sound a guitar makes from other instruments such as a piano. The fundamental frequency is the musical note.