MOMENTS OF PARALLEL FORCES
(TORQUE)

OBJECTIVE: To become better acquainted with the principles of moments (torques), with the second condition for equilibrium, and with the resolution of parallel forces.

THEORY: The FIRST condition for equilibrium states that the horizontal and vertical components of all forces acting on a body must add up to zero:

$$\Sigma F_x = 0 , \quad \Sigma F_y = 0$$

The SECOND condition for equilibrium states that all the moments (or torques), τ, about some axis must also add up to zero:

$$\Sigma\tau = 0 .$$

A moment of a force, or the torque, is defined as the product of the force by its moment arm (that is, the perpendicular distance from the line of action of the force to the axis as shown in Fig. 1):

$$\tau = F\, r \sin\theta$$

where θ is the angle between the direction of ***r*** and ***F***. By convention, counterclockwise moments are assigned a positive sign, while clockwise moments are given a negative sign. This can be reversed if it is more convenient as long as you are consistent. When an object is in equilibrium therefore, the counterclockwise moments must equal the clockwise moments. This is another way of stating the second condition for equilibrium.

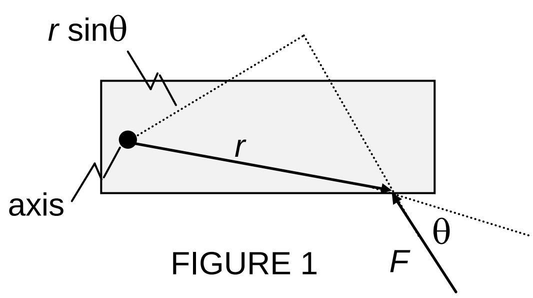

FIGURE 1

In this experiment, we will use a meter stick balanced in a horizontal position so that the length of the moment arms may be calculated using positions obtained directly from the meter stick. The forces will be due to gravity and hence will be vertical, so the **angle θ will always be 90°** [and hence, $\sin\theta$ will be 1]. There is likely to be some slight differences between the calculated values and the experimental values due to friction, discrepancies in the weights used, and difficulty in reading the meter stick with great accuracy.

Part 1: Weight and the Center of Gravity

PROCEDURE:

The mass and center of gravity of the meter stick itself must first be determined before we can proceed with more complicated cases. If the meter stick is symmetric, then we would expect the stick to balance in the middle, i.e., at the 50 cm mark. However, in our case, we have placed weights in the stick so it is no longer symmetric and will not balance at the 50 cm mark.

a) The **experimental center of gravity** is found by balancing the meter stick on the vertical support by adjusting the location of the plain knife edge (the fulcrum). **Find and record** the position of the center of gravity of your particular weighted meter stick. Be sure to read the location of the fulcrum correctly – it is directly across from the "wings" that stick out to balance on the big black part. You will need this location again in parts 2 and 3, so **be sure to record it** where you can find it later.

NOTE:* *In all the following procedures, whenever a mass is hung from the meter stick, the mass of the knife-edged hanger must be added to that of the hanging mass and this total used in the calculations.* ***Weigh each of the hangers using the digital scale and record their respective masses.* *Be sure to know the correct mass of a hanger when you use it.*

b) We can use the fulcrum and known weights to find an experimental value for the (as yet) unknown weight (and mass) of the stick. We'll check later with a scale to see if this actually works. *Note: all of the forces in this experiment will be due to gravity. This means that all forces will be in the form of F=W=mg. Since there will be a "g" in each force and so in each torque, we can cancel out the g constant and simply work with the masses instead of the weights.*

To do this:

1. Move the fulcrum from the center of gravity location of part a above to a new location at the 55 cm mark. The stick will no longer be balanced.
2. Then place a hanger loaded with 100 gm plus hanger at a position such that the meter stick is balanced.
3. Record this location.
4. Using this data, calculate the mass (since we cancelled the g in the weight) of the meter stick – you can do this using the fact that the net torque is zero on the stick if the stick is balanced. HINT: Where is the weight (mass) of the meter stick considered to be located? In this case it is easiest to measure the moment arms from the locations of each of the two weights (the weight of the hanging mass and the weight of the stick) to the location of the fulcrum. Note: if we measure the moment arms from the fulcrum, then even though the fulcrum provides a force, it's moment arm will be zero and hence the fulcrum will provide no torque. From this information **calculate the mass of the meter stick**.

c) Now move the fulcrum **to 60 cm** and then **to 65 cm**, EACH TIME calculate the mass of the meter stick.

d) Average your three values for the mass of the stick. QUESTION: How close are the three values to one another? QUESTION: Should they have been the same? Now measure the mass of the stick using the scale and compare with your average mass. [In the calculations for Part 3, use the scale reading for the mass of the meter stick and the center of mass determined above for the location of the mass of the stick.]

Part 2: Statics and Torque

To further check the Principle of Moments ($\Sigma\tau = 0$ when balanced), place the fulcrum at the original center of gravity of the stick (from part 1a above). As long as we determine the moment arms as the distance of each applied weight to the fulcrum, this will then ELIMINATE the torque due to the fulcrum and the torque due to the weight of the meter stick since both will be directed through the fulcrum and hence have a zero moment arm and so will have zero torque.

In each of the four cases below, determine either the moment arm or the mass **by calculation** using $\Sigma\tau = 0$ and then **compare** to the value you get **from direct measurement** using the apparatus.

a) Hang 200 gm plus hanger at a distance of 15 cm from the fulcrum. Balance this with 100 gm plus hanger on the opposite side. Determine the moment arm of the 100 gm mass by calculation using theory, that is, using $\Sigma\tau = 0$. Measure the experimental moment arm using the apparatus. Compare your calculated theoretical value to your measured experimental value.

b) Hang 200 gm plus hanger 20 cm to the left of the fulcrum and 100 gm plus hanger 35 cm also to the left of the fulcrum. Balance this with 200 gm plus hanger on the right side. Record the location of the right side mass. Determine the experimental moment arm of the right side mass. Using the $\Sigma\tau = 0$, calculate the theoretical value of the right side moment arm. *Question: How do you work with three torques?* Compare your experimental value to your theoretical value of the moment arm for the right side mass.

c) On the right, hang 200 gm plus hanger 15 cm from the fulcrum. On the left, hang 500 gm plus hanger 15 cm from the fulcrum and 100 gm plus hanger 30 cm from the fulcrum. Achieve balance by hanging 200 gm plus hanger somewhere on the right. Again compare calculated theoretical AND experimental measured values of the moment arm.

d) Determine the mass of the lead piece by hanging it 25 cm from the fulcrum and balancing it with 100 gm plus hanger on the other side. Here, take the experimentally determined moment arm as correct, and calculate the theoretical mass of the lead piece. Compare the mass with that read from a scale (the experimental value).

Part 3: Determining the Balance Point: Calculation and Experiment

In part 1a, we found by experiment where the center of gravity of the meter stick was. We didn't know where the extra weights were placed or how heavy those extra weights were. In this third part, we will load up the stick with weights placed at known positions, and then try to determine **where to put the fulcrum**, i.e., we will try to determine where the center of gravity of the loaded stick is. We will do this both by theory using the Principle of Moments and by actual experiment. We will then see how well the theory agrees with the actual experiment.

THEORY: Parallel forces can be balanced by a single equalizing force, E, in the antiparallel direction. This is similar to what we did in the first experiment on Concurrent Forces, but now is expanded to include torques. The value of this FORCE is determined by the FIRST condition of equilibrium, $\Sigma F_y = 0$. There will be only y-components involved. The point of application (x) of this equilibrant (E) can then be determined by using the SECOND condition for equilibrium, $\Sigma\tau = 0$. Since we don't know where the fulcrum will be (x), it will be hard to measure moment arms from that position like we did in Parts 1 and 2 above. Here it will be more convenient to take moments about the **zero end** of the meter stick. QUESTION: *Why is this allowed?*

As an example, consider Fig. 2.

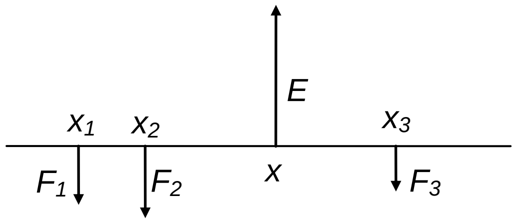

FIGURE 2

Experimentally, weights are hung at various places, and then the fulcrum is moved to achieve balance. *Note: since the fulcrum will no longer be located at the center of gravity of the stick (location determined in part 1a above), the weight of the meter stick (also determined in Part 1) acting at its center of gravity must be taken into account and treated as one of the forces.*

Applying the first condition of equilibrium, $\Sigma F_y = 0$:

$$E - F_1 - F_2 - F_3 = 0 \ . \qquad (1)$$

Applying the second condition of equilibrium, $\Sigma\tau = 0$:

$$F_1 x_1 + F_2 x_2 + F_3 x_3 = E x \ . \qquad (2)$$

Solving Eq.(1) for E and using this in Eq.(2), we can solve for x:

$$x = (F_1 x_1 + F_2 x_2 + F_3 x_3) / (F_1 + F_2 + F_3) \qquad (3)$$

This then locates the position, x, of the equilibrant, E, which in the experiment is supplied by the fulcrum.

PROCEDURE:

a) Hang 200 gm plus hanger **at** the 20 cm mark, and 100 gm plus hanger **at** the 80 cm mark, and obtain balance by moving the fulcrum. Using the above equations, determine the theoretical value for the location of the fulcrum, x, and compare with the result of your experiment.

b) Hang 200 gm plus hanger at the 10 cm mark, 100 gm plus hanger at the 25 cm mark, and 200 gm plus hanger at the 70 cm mark, and again obtain balance by moving the fulcrum. Again, determine the theoretical position of the equilibrant and compare to the result of your experiment.

REPORT:

For each experimental procedure you performed, answer any QUESTIONS, report your experimental findings, and compare them with your calculated predictions. Discuss the major sources of error that would account for your discrepancies.