

COMPOSITION OF CONCURRENT FORCES

OBJECTIVE: To see if the result of applying three forces on an object can be determined by ADDING the three forces as VECTORS.

GENERAL PROCEDURE: In this experiment your lab instructor will assign you one set of three vectors from each of the two groups listed on the fourth page. For each of these two vector sets, add the three vectors to get the result using all three methods below (Parts 1,2,3). Be sure to estimate your uncertainty in any experimental measurement. Part 4 will lead you through a method of determining the overall uncertainty of your results. The last two pages will show you an **example of a written report** for this experiment.

Part 1: Analytical Method

THEORY: To add two or more vectors together, it is first necessary to express each of the vectors in **rectangular** components, (A_x, A_y) . If a vector is expressed in **polar** form (A, θ) , where A is the magnitude and θ is the angle indicating direction, then the rectangular components can be found using the basic trigonometry of a right triangle:

$$\begin{aligned} A_x &= A \cos(\theta_A) & A_y &= A \sin(\theta_A) & (1) \\ \text{and similarly, } B_x &= B \cos(\theta_B), C_x = C \cos(\theta_C) & B_y &= B \sin(\theta_B), C_y = C \sin(\theta_C) \end{aligned}$$

Then all the x-components of the vectors are added together to obtain the resultant x-component, R_x ; all the y-components are added to obtain the resultant y-component, R_y ; and if in 3-D, the same goes for the z-components:

$$R_x = \Sigma A_x = A_x + B_x + C_x \quad \text{and} \quad R_y = \Sigma A_y = A_y + B_y + C_y \quad (2)$$

Once the rectangular components of the resultant vector have been obtained, the magnitude R of the resultant vector can be obtained using the Pythagorean Theorem:

$$R = [R_x^2 + R_y^2]^{1/2} \quad (3)$$

The angle that the resultant vector makes with the x-axis, θ_R , can be determined using the basic trigonometry of a right triangle:

$$\theta_R = \tan^{-1} (R_y / R_x) \quad (4)$$

Be careful: If R_x is negative, θ_R is in the 2nd or 3rd quadrant. If the \tan^{-1} function returns an angle in the 1st or 4th quadrant when R_x is negative, **then** you will need to add 180° to the angle θ_R so that it is in the correct quadrant.

PROCEDURE:

Add each of the two sets of three vectors analytically. Express the resultant vectors in polar form, i.e. find the magnitude and angle of each of the resultant vectors.

Part 2: Graphical Method

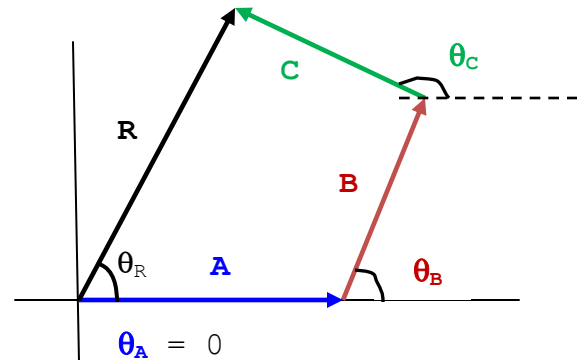
THEORY: To add two or more vectors together graphically, it is first necessary to set up a coordinate system. For vectors expressed in polar coordinates, an origin and a horizontal line are needed. Then each vector can be expressed as a directed line segment on this graph. The length of the line is determined by the magnitude of the vector according to some scale. The direction of the line is determined by having the line segment directed at an angle θ measured counterclockwise from the horizontal coordinate line (or from some line parallel to the horizontal coordinate line) where θ is equal to the angle of the vector. The first vector starts at the origin. The next vector starts at the tip of the first vector. And each additional vector to be added starts at the tip of the preceding vector. The resultant is then obtained by drawing in a line segment from the origin (tail of the first vector) to the tip of the last vector. The length of this segment then gives the magnitude of the resultant vector, and the angle measured from the horizontal axis gives the angle of the resultant vector.

PROCEDURE:

Add each of the two sets of three vectors graphically.

Warning: placing the origin in the middle of the paper is not always the best - you may run out of room on the paper! Use a scale of **1 cm for 20 grams**.

Estimate how accurately you can draw and measure each length and angle.



Part 3: Experimental Method (Using the Force Table)

All of our forces are going to be due to gravity with the pulleys changing the direction of the gravitational force from down to horizontal directed along the line where the pulleys are located.

THEORY: The three forces representing the three vectors to be added are applied to the ring on the force table by setting a pulley on the force table at the angle corresponding to each of the three vectors and then attaching weights (masses) corresponding to the magnitude of each vector to strings hung over the pulleys. The angle and weight (mass) needed to balance the system and keep the ring centered on the post determine a fourth vector, **E**, such that all four vectors add to the zero vector:

$$\mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{E} = \mathbf{0} \quad (5)$$

or,

$$\mathbf{A} + \mathbf{B} + \mathbf{C} = -\mathbf{E} = \mathbf{R} \quad (6)$$

Therefore the resultant vector, **R**, is equal in magnitude to **E** but oppositely directed. Thus the angle determined on the force table is 180° different from the angle of the resultant vector.

PROCEDURE:

Add each of the two sets of three vectors using the force table. NOTE: The weight holder itself has a mass of 50 grams. Should this be included in your weight (mass) determination? Estimate how well you can set and measure the angles, and estimate how well you can determine the weight (mass) of the fourth vector **E**.

Part 4: Experimental Uncertainty

THEORY: To begin this process of determining experimental uncertainty for the results for this experiment, let's consider comparing your experimental results from the graphical method with the theoretical results from the analytical method. First, what did you measure and how well were you actually able to measure it? Second, were there any other aspects of the experiment that might lead to uncertainties or errors? Third, can you estimate how much all of these uncertainties or errors affect your results?

To help you in this estimating uncertainty process, we will employ a "brute force" method of determining how good your experimental results are. This "brute force" method simply calculates what the final results might be using values (in this case the three magnitudes and three angles) that are different by the estimated amount of uncertainty. For example, if the uncertainty in your angle measurements was 1° , then we would add 1° to all three angles and recalculate the resultant force. But of course, we also need to see what the result would be if we subtracted 1° from all three angles. Even this would not give us all the possibilities since we might have been off by -1° on the first angle, $+1^\circ$ on the second, and $+1^\circ$ on the third. In fact, there are eight combinations of adding or subtracting 1° from three angles: (+++, ---, +--, +-+, -+-, -+-, ---). In addition, you might have been off by 2 mm (= 4 grams) on each of your three length measurements. This would also give eight different possible combinations for calculating a possible result. Finally, we would have to consider both possibilities at once: error in angle and error in length. This would entail doing $8 \times 8 = 64$ calculations. This is a lot of calculations to do, but fortunately we have a computer that is well suited to do this type of repetitive operation.

PROCEDURE: If the computer is off, turn it on. Click on the Concurrent Forces icon. When asked to enter in values, use the values from one of the two problem sets that you did. This program will then calculate all 64 possibilities and so will allow you to see what range of values for the resultant force (both magnitude and angle) you might expect for your uncertainties in measurements.

We leave the analysis of the force table experiment to you. Be sure to analyze the results using this same procedure, but do not worry about employing the brute force method here. Simply make the best estimates you can.

REPORT:

Compare the results of all three methods of adding three vectors together. Do the results agree within experimental error? How big are the experimental errors? How big is the resultant experimental uncertainty for your result?

Note on **units**: Grams is a unit of mass, not force. Since the force of gravity depends on the mass and the earth (we'll see in Part 2 of the course that $F_{\text{gravity}} = W_{\text{weight}} = mg$), and since all of our forces are due to gravity on the earth, we can simplify our work by eliminating the constant due to the earth (that is, cancel out the g factor) and treat all of our forces as if they were equal to our masses in grams.

VECTOR TABLE

		A	B	C
	SET	(Mag,Angle)	(Mag,Angle)	(Mag,Angle)
Group I	1	(200 gm, 0°)	(100 gm, 70°)	(100 gm, 135°)
	2	(200 gm, 0°)	(150 gm, 80°)	(100 gm, 140°)
	3	(150 gm, 0°)	(100 gm, 45°)	(150 gm, 120°)
	4	(100 gm, 0°)	(200 gm, 80°)	(150 gm, 135°)
	5	(150 gm, 0°)	(200 gm, 60°)	(150 gm, 150°)
Group II	6	(200 gm, 0°)	(150 gm, 300°)	(100 gm, 190°)
	7	(200 gm, 0°)	(100 gm, 135°)	(150 gm, 280°)
	8	(150 gm, 0°)	(200 gm, 140°)	(150 gm, 200°)
	9	(100 gm, 0°)	(150 gm, 330°)	(200 gm, 170°)
	10	(100 gm, 0°)	(200 gm, 200°)	(150 gm, 135°)

(see the next two pages for an example of a formal written report for this lab)

Sample of a Written Lab Report

(You should use this as a guide when writing up your formal written lab reports.)

Object: To see if the result of applying three forces on an object can be determined by ADDING the three forces as VECTORS. *(Note that the original objective was used here.)*

Data: Our group was assigned problems 5 and 6. For these problems, the three vectors were:
 Problem 5: $A = (150 \text{ gm}, 0^\circ)$, $B = (200 \text{ gm}, 60^\circ)$, and $C = (150 \text{ gm}, 150^\circ)$
 Problem 6: $A = (200 \text{ gm}, 0^\circ)$, $B = (150 \text{ gm}, 300^\circ)$, and $C = (100 \text{ gm}, 190^\circ)$.

The first method, the analytical method, employed only calculations. No data were taken.

The second method, the graphical method, had us create graphs. See the attached graphs that we made. *(No graphs were made for this example.)*

The third method, the force table method, gave the following results:

Problem	Vector A	Vector B	Vector C	Vector E (the equilibrant)
5	150 gm at 0°	200 gm at 60°	150 gm at 150°	$280\text{g} \pm 10\text{g}$ at $245^\circ \pm 1^\circ$
6	200 gm at 0°	150 gm at 300°	100 gm at 190°	$230\text{g} \pm 10\text{g}$ at $141^\circ \pm 2^\circ$

(Note that a table was used for the data. This is highly recommended!)

Graphs: See the attached graphs showing the graphical method of adding vectors. *(These graphs are not included in this sample report. Usually, data is recorded first and then put in graphs to visually show the relationships. Spreadsheets such as Excel provide a powerful and easy way of graphing data. Be sure to show all data points, and also include a best fit line. Include the equation of the best fit line, but also convert it from the $y(x)$ format into a physics format that includes units for all values.)*

Calculations: *(Only one sample calculation for each type should be shown. Units should always be included in all calculations.)*

For the first, analytical, part, we have to first convert all the vectors from the given polar form into rectangular form:

Vector A: 150 gm at 0° : $R_A = 150 \text{ gms}$; $\theta_A = 0^\circ$

$$A_x = R_A \cdot \cos(\theta_A) = 150 \text{ gm} \cdot \cos(0^\circ) = 150 \text{ gm}; \quad B_x = 100.0 \text{ gm}; \quad C_x = -129.9 \text{ gm}$$

$$A_y = R_A \cdot \sin(\theta_A) = 150 \text{ gm} \cdot \sin(0^\circ) = 0 \text{ gm}. \quad B_y = 173.2 \text{ gm}; \quad C_y = 75.0 \text{ gm}$$

Next we have to add all of the x components to get the total x component:

$$X_R = A_x + B_x + C_x = 150.0 \text{ gms} + 100.0 \text{ gms} - 129.9 \text{ gms} = 120.1 \text{ gms}$$

$$Y_R = A_y + B_y + C_y = 0.0 \text{ gms} + 173.2 \text{ gms} + 75.0 \text{ gms} = 248.2 \text{ gms}$$

Finally, we convert from the rectangular form back to the polar form:

$$R = [X_R + Y_R]^{1/2} = [(120.1 \text{ gms})^2 + (248.2 \text{ gms})^2]^{1/2} = 275.73 \text{ gms}$$

$$\Theta = \tan^{-1} [Y_R/X_R] = \tan^{-1} [248.2 \text{ gm} / 120.1 \text{ gm}] = 64.18^\circ.$$

(Since both the x components and the y components are positive, the angle should be in the first quadrant, and it is.)

For the second, graphical, part, we have to first convert the magnitude from grams to centimeters. To do this, we will use the scale factor of 20 grams = 1 cm.

$$R_A = 150 \text{ grams} * (1 \text{ cm} / 20 \text{ grams}) = 7.5 \text{ cm}; \quad R_B = 10.0 \text{ cm}; \quad R_C = 7.5 \text{ cm}.$$

For the third part, we have to determine the resultant vector, R, from the equilibrant vector, E:

$$\Theta_R = \Theta_E \pm 180^\circ.$$

$$\Theta_R = \Theta_E \pm 180^\circ = 245^\circ \pm 180^\circ = 65^\circ.$$

Results: (Often a table is the best way to show results.)

problem	Analytical	Graphical	Force table
5	275.73 gm @ 64.18°	Type in results From graph here	280g ± 10g at 65° ± 1°
6	229.88 gm @ -39.84°	Type in results From graph here	230g ± 10g at -39° ± 2°

Sources of Experimental Uncertainty:

In the analytical part, the only source of uncertainty is the round-off error in the calculator that will be much smaller than any experimental error.

In the graphical part, there are two main sources of experimental error. We can only measure out lengths to within about 1 mm, which leads to an uncertainty in drawing the length of each vector of 2 grams. We can only measure angles to about 1°.

In the force table part, we can only determine the magnitude of the equilibrant vector to within 10 grams since we have no smaller weights available. Due to the strings tied to the central ring, the strings can only be lined up correctly to about 1°.

Conclusion and Discussion:

As you can see from the table of results, our Force Table results were within about 5 grams and within 1 degree of matching the analytical results. Both the magnitude and the angle match up within the range of uncertainty. (You should say a similar thing about how well the graphical matches the analytical and the Force Table results.)

This indicates that we can add force vectors just like we can add position vectors: by breaking the force vectors into components and adding the x components together and adding the y components together to get the result in rectangular components. We can then convert back to polar form since that is usually the nicest way to express our result.

(Note that this refers back to the original objective for the lab.)