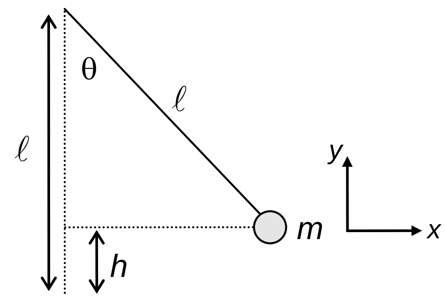
OSCILLATIONS

OBJECTIVE: To study the oscillations of a pendulum and to determine what affects the period, T, of the pendulum.

THEORY: The period of the pendulum is the time it takes for the bob to go from some initial θ_o through $\theta = 0$, up to some negative θ, and then back through $\theta = 0$ until it finally reaches its original value, θ_o. So to determine the period, T, we first need to determine how θ behaves with time, i.e., we need θ as a function of time, $\theta(t)$. This problem can theoretically be attacked either from (a) Newton's Second Law ($\Sigma\vec{F} = m\vec{a}$, $\Sigma\tau = I\alpha$) or from (b) Conservation of Energy.

FIGURE 1



(a) **Newton's Second Law** requires that we look at all the forces acting on the pendulum. Consider Fig. 1. There is gravity ($W = mg$) acting down, there is the tension in the string (T_s) acting along the direction of the string, there is air resistance acting in the direction opposite the velocity ($R = -bv$ 2 is one possible model), and there may be friction at the top of the support (which we neglect here). This is a two-dimensional problem, so:

$$\Sigma F_x = -T_s \cos(90° - \theta) - b\,v_x^2 = m\,a_x \qquad (1x)$$

$$\Sigma F_y = +T_s \sin(90° - \theta) - mg - b\,v_y^2 = m\,a_y \qquad (1y)$$

These equations are not easy to solve in this form. We can simplify these equations if we choose to ignore air resistance (just set b=0 in the above equations). But we still have the problem of how to deal with θ since θ depends on x and y [from the geometry we see that $\theta = \tan^{-1}(x/y)$] which is part of the argument of the sine and cosine functions.

A better approach might be to note that, if we assume the string does not break or stretch, then this condition can be stated as $x^2 + y^2 = \ell^2$ and this leads to a circular motion type problem involving the single variable, θ, instead of the two variables x and y. We can write that $v = \ell\omega$ where ω is the angular velocity of the bob. If we again neglect any frictional effects at the support but keep the air resistance term, we can then analyze the rotation of the pendulum in the presence of the torques provided by gravity and air resistance. The air resistance force is perpendicular to the string's direction since it is opposite the velocity direction. The lever arm of its torque is thus ℓ. Gravity provides a torque with a lever arm of $\ell\sin\theta$. Thus, Newton's Second Law gives

$$\Sigma\tau = -mg\,\ell\sin\theta - b\,v^2\ell = I\alpha\,.$$

$$-mg\,\ell\sin\theta - b\,\ell^3\omega^2 = I\alpha \qquad (2)$$

(Again, we can set b=0 if we wish to ignore air resistance.)

Since $\omega = d\theta/dt$ and $\alpha = d\omega/dt = d^2\theta/dt^2$, we obtain a second order differential equation for $\theta(t)$:

$$-m g \ell \sin\theta - b \ell^3 (d\theta/dt)^2 = I (d^2\theta/dt^2) \qquad (3)$$

Note that θ is inside the argument for the sine function. The solution of this differential equation is beyond the math background assumed for this course.

(b) **Conservation of Energy** requires that we consider all the forms of energy in the system: potential energy due to gravity: mgh, kinetic energy: $\frac{1}{2} m v^2$, and any energy lost to friction. Referring to Fig. 1, $h = \ell - \ell \cos\theta$, $v = \omega \ell$ (circular motion with ℓ acting as the radius), and E_{lost} depends on the angular speed, ω, in a non-trivial way which depends on the integral of the frictional force over the distance traveled (or the integral of the frictional torque over the angle swept out):

$$mg\ell [1 - \cos\theta_i] + \frac{1}{2} m \omega_i^2 \ell^2 = mg\ell [1 - \cos\theta_f] + \frac{1}{2} m \omega_f^2 \ell^2 + E_{lost} \qquad (4)$$

If we ignore air resistance, we can drop the E_{lost} term. But even this simplifying assumption does not make this equation simple since θ is in the argument of the cosine function; and since $\omega = d\theta/dt$, this gives us a first order differential equation for θ(t) whose solution is also beyond the math background assumed for this course. But if we do ignore friction, it does allow us to draw an energy plot of potential energy and total energy vs. θ to try to find a qualitative description of the motion.

In both cases, the theoretical analysis does not give us an easy way of calculating the period.

Part 1: Finding an experimental relationship

PROCEDURE:

You are provided with a support, string, masses to be used as the pendulum bob, a protractor and a stop watch.

a) Make different pendulums and determine the period of each. Questions to consider:

(1) Where do you measure the length of the pendulum from: from the top of the bob, the center of the bob, the bottom of the bob, or the center of gravity of the bob?

(2) Should you make one timing of one oscillation, make several timings of one oscillation, make one timing of several oscillations, or make several timings of several oscillations?

(3) Does resistance play a significant part in the oscillations? Does resistance cause the same or different effects for different pendulums? Can you explain this? HINT: see how the angle gets smaller after successive oscillations and record your results.

(4) What are the major sources of experimental uncertainty in your measurements?

Suggested Parameter Values

a-1) Measure the period as a function of mass with length and starting angle as constants. Set the length to about 1.0 meter (does not have to be exactly 1.0 m, but you do need to measure the length used for each mass and it should be nearly the same for each) and use a starting angle of 25°. Measure the periods for masses of 0.500 kg, 0.200 kg, 0.100 kg, and 0.020 kg.

a-2) Measure the period as a function of starting angle with length and mass as constants. Again set the length to about 1.0 meter (record the actual value used) and here use a mass of 0.200 kg. Measure the periods for starting angles of 10^o, 25^o, 45^o, and 70^o.

a-3) Measure the period as a function of length with mass and starting angle as constants. With a mass of 0.200 kg and a starting angle of 25^o, measure the periods for lengths of 1.20 m, 0.80 m, 0.40 m, and 0.20 m.

a-4) In order to get an estimate of the air resistance, use a light mass of 20 gm with a 1.0 m length and an initial angle of 70^o. After five complete swings observe the maximum angle (amplitude); the result will almost certainly be less than the starting angle. This result can be used in the numerical work of Part 2, b). Do this also using a heavier 500 gram mass.

b) See if you can experimentally determine how the period of the pendulum depends on the various parameters that you can control [e.g., you can control the mass of the pendulum bob, but you cannot control the gravity in the room]. HIn other words, can you determine an equation for the period, T, in terms of m, L, and θ? HINT: make graphs of *T* vs each parameter you test. To see the relative sizes of the effects that each parameter has on the period, make your *T* axis identical for each plot.

c) By analyzing your data, **design** a pendulum with a period of (1) 0.1 sec., (2) 1.0 sec., (3) 10.0 sec. If possible, check your predictions by making a pendulum with each of these periods.

Part 2: Numerical Analysis

a) We will use the computer in this part. If the computer is not on, turn it and the monitor on. Click on the OSCLAB icon, and follow the on-screen directions. Set *b*=0 for now. [We will see it's effect below in part b.] Enter your predicted values for m, ℓ, and θ_o that should give a period of 1.0 second for the pendulum. Once you see how well the computer predicts the results of your experiment, determine the numerical prediction for the period of each of your other two designs for Part 1(c) above, i.e., for the 0.1 second pendulum and the 10 second pendulum.

NOTE: The numerical method works by entering in the initial conditions (here: m,l,θ_o, ω_o and b), and choosing a small time interval Δt. The time interval should be much smaller than the expected period, T. The program assumes that $\omega_o = 0$. From these values, the computer uses Eq. (2) to calculate α, then assumes this α is constant over the Δt and calculates a new ω, then a new θ, and then updates the time. With the new values of θ and ω, the program iterates the above procedure. The computer recognizes a period has passed when ω changes sign the second time.

b) Using the numerical routine above, determine a value for *b*, the air resistance coefficient, by watching how the angle deteriorates over repeated periods: that is, try to match your experimental observations of how the maximum angle of the pendulum decreased after five swings from part 1 a-4 above with your numerical results from trying different values of *b* in the numerical routine that predicts the value of the period (and also the angle at the end of each period). If you are successful in getting one value of *b* that works for one mass, length and initial angle, see if the same value of *b* will also work for a different mass, length and initial angle. Does this suggest that the value of *b* depends on any or all of the parameters of the system?