Newton's Second Law
(without friction)

OBJECTIVE: To consider various forces and how they affect the motion of objects, particularly the motion of a cart moving along a ramp.

THEORY: One of the fundamental laws of nature is that forces do NOT cause motion, $\Sigma F \neq mv$, but rather that forces cause **changes** in motion (accelerations),

$$\Sigma \mathbf{F} = m\mathbf{a} \ .$$

Notes:
1. The change in motion is due to the **SUM of the forces**, not to each individual force.
2. Since force and acceleration are vectors, this is a **vector equation** which means that we must break the vectors into rectangular components. In this experiment we will be dealing with vectors and motion in only 2-D rather than the full 3-D (actually, we are dealing in 3-D but all of our forces and all of our motion will only be in 2-D). For this we can use either *x* and *y* rectangular components, or we can use // and ⊥ (parallel and perpendicular) components.
3. In this experiment we will be dealing with **two objects** (the cart and the balancing mass) as shown in Fig. 1, rather than with just one object. In this case we will apply Newton's Second Law to each object individually. We will then have to consider how these two objects and their motions are related.

In this experiment we will deal with three kinds of force: weight, W; contact force (also called the normal force), F_c; and tension, T.

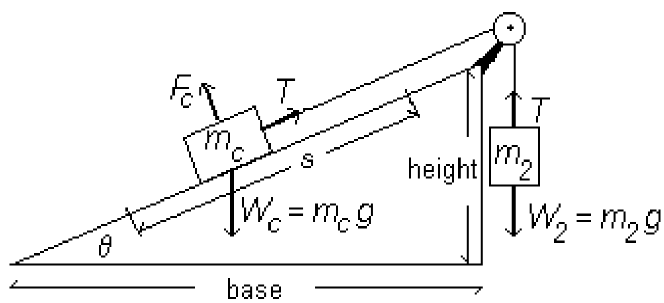

FIGURE 1

1. Consider the **cart** first: $\Sigma F_{on\ c} = m_c a_c$; Since the cart will only accelerate in the parallel direction and not the perpendicular, we will choose to use the parallel and perpendicular components, with $a_{c\text{-}//} = a_c$ and $a_{c\text{-}\perp} = 0$. There are three forces acting on the cart: the contact force, F_c, which acts strictly in the perpendicular direction; the tension in the string, T, which acts strictly in the parallel direction as long as we align the string pulling the cart parallel to the ramp; and the weight of the cart, $W_c = m_c g$, which acts down. We therefore need to break the weight into // and ⊥ components. From geometry we get: $W_{c\text{-}//} = m_c g \sin\theta$, and $W_{c\text{-}\perp} = m_c g \cos\theta$. Putting everything together, we have:

$$\Sigma F_{c\text{-}//} = T - m_c g \sin\theta = m_c a_c \ ; \text{ and} \qquad (1)$$

$$\Sigma F_{c\text{-}\perp} = F_c - m_c g \cos\theta = 0 \ . \qquad (2)$$

The above two equations involve 3 unknowns: T, a_c, and F_c. Thus we need more info to solve this situation.

2. Next, consider the **balancing mass, m_2**. Since it only can move up and down, we have in this case a 1-D situation. We also have only two forces: the tension in the string, *T*, which is the same tension as is on the cart; and the weight of the balancing mass, $W_2 = m_2 g$. Newton's 2nd law for this mass is: $T - m_2 g = m_2 a_2$. As far as the acceleration goes, we note that when the cart accelerates up the ramp, the balancing mass accelerates down, so we can say: $a_2 = -a_c$. Hence, Newton's Second Law for this mass gives:

$$\Sigma F_2 = T - m_2 g = m_2(-a_c) . \qquad (3)$$

3. We now note that Eqs. (1) and (3) give us two equations for two unknowns: (*T* and a_c). The third equation, Eq. (2), allows us to find F_C, but we don't actually need F_C. If we eliminate *T*, that is, solve Eq. (3) for T in terms of a_C and use this in Eq. (1), we have for the theoretical acceleration of the cart, $a_{c\text{-}th}$:

$$a_{c\text{-}th} = g\,(m_2 - m_c \sin\theta) / (m_2 + m_c) \qquad . \qquad (4)$$

We can now solve for the "balancing mass", $m_{2\text{-}balancing}$, necessary for $a_{c\text{-}th}$ to be zero so that the block does not move:

$$\mathbf{m_{2\text{-}balancing} = m_c \sin\theta.} \qquad \mathbf{(5)}$$

We can now use Eq. (5) to adjust Eq. (4) to read:

$$\mathbf{a_{c\text{-}th} = g\,(m_2 - m_{2\text{-}balancing}) / (m_2 + m_c)} \qquad . \qquad \mathbf{(6)}$$

Since this acceleration will be constant as long as the angle is constant, we can use the equations for constant acceleration:

$$v = v_o + at \qquad (7)$$

and

$$s = s_o + v_o t + \tfrac{1}{2}at^2 \ . \qquad (8)$$

If we start at a location we call zero, then $s_o = 0$ m; if we release the cart, then we know that $v_o = 0$ m/s. This simplifies equation 8 to become:

$$\mathbf{s = + \tfrac{1}{2}at^2} \qquad \mathbf{(8a)}$$

By calculating the theoretical acceleration, $a_{c\text{-}th}$, from **Eq. (6)** and measuring the distance, s, we can then use **Eq. (8a)** to **predict the time, t_{th}** , for the cart to go up the ramp.

PROCEDURE:

We'll investigate two situations: where the cart doesn't move (static equilibrium), and where it does move (dynamic). We'll test the validity of the theory developed above in both situations.

Statics (no movement) **Parts 1-4**

1. (a) **Set the ramp** at an angle of 25° by adjusting the height of one end of the ramp above the table by adjusting the height of the supporting crossbar. Hint: $\sin(\theta)$ = opposite/hypotenuse. The distance from the table end of the ramp to a yellow mark on the ramp near the top is 1.00 meters. Be sure to measure the height of the top end from the **bottom side** of the ramp since that is what touches the table.
(b) **Estimate** the uncertainty in your angle, $\delta\theta$, due to the uncertainties of the values you use to try to make the angle 25° such as δh. We'll use the symbol δ to indicate an uncertainty amount.
(c) **Weigh** the cart on a scale and record its mass, m_c. Be sure the scale is measuring in grams.
(d) **Attach** a string to the cart and pass the string over the pulley and tie a loop if there isn't one already for hooked weights.
(e) **Find** the mass hanging over the pulley that balances the cart so that it neither rolls up the ramp nor rolls down the ramp (the cart being initially at rest); call this mass m_{25}.
(f) **Determine** how much mass can be added or subtracted and still keep the cart from rolling: we'll call this experimental uncertainty δm_{25}.

1-1. **Record** m_c, m_{25}, δm_{25}, and the data used to get the ramp angle at 25°.

1-2. From Newton's Second Law, **calculate** the balancing mass from theory - **Eq. (5)**.

1-3. **See** if the theoretical balancing mass from step 1-2 above falls within the experimental range of $m_{25} \pm \delta m_{25}$.

2. Keep the setup from Step 1 with the following exception. **Weigh** the black block and **place it** on the cart. The mass of the cart is now $m_{c+} = m_c + m_{block}$. Record m_{block} and m_{c+}. **Repeat** the rest of Step 1 (that is, find the new balancing mass, m_{25+}, and δm_{25+}).

2-1. Since the mass of the cart approximately doubles by adding the external mass, (i.e., $m_{c+} = m_c + m_{blbock} \cong 2m_c$), ***does the new balancing mass, m_{25+} (and hence weight) also approximately double (i.e., does $m_{25+} \cong 2 \cdot m_{25}$)?***

2-2. ***Does theory predict that the balancing mass should double?***

2-3. ***Does your experimental uncertainty δm_{25+} also double?***

2-4. **Calculate** the theoretical balancing mass from **Eq. (5)** and **see** if it is within the experimental range $m_{25+} \pm \delta m_{25+}$.

3. **Remove** the black block so the mass of the cart is just the original m_c. Now **change the angle** from 25° up to 50°. (This doubles the angle.) **Find** the hanging mass that balances the cart on the steeper ramp; call this mass m_{50} . **Determine** how much weight can be added or subtracted and still keep the cart from rolling: call this δm_{50}.

3-1. **Record** m_c, m_{50}, δm_{50}, and the data used to get the ramp angle at 50°.

3-2. ***Since the angle is doubled, does the balancing mass, m_{50}, also double?***

3-3. ***Does theory say the balancing mass should double?***

3-4. **Calculate** the theoretical balancing mass from **Eq. (5)** and see if the theoretical value falls within the experimental range $m_{50} \pm \delta m_{50}$.

4. Now **change the angle** from 50° down to 12.5°. (This angle is one-half of the initial 25°.) Use the cart without the external mass (without the black block). **Find** the hanging mass that balances the cart on the less steep ramp; call this mass $m_{12.5}$. Determine how much mass can be added or subtracted and still keep the cart from rolling: call this $\delta m_{12.5}$.

4-1. **Record** $m_{12.5}$, $\delta m_{12.5}$, and the data used to get the ramp angle at 12.5°.

4-2. ***Since the angle is half of the original, is the balancing mass also half of the original?***

4-3. ***Does theory say the balancing mass should be half?***

4-4. **Calculate** the theoretical balancing mass using **Eq. (5)** and see if the theoretical value falls within the experimental range $m_{12.5} \pm \delta m_{12.5}$.

Dynamics (accelerated motion) **Parts 5-6**

5. Reset the angle to 25°. **Add** between 30 and 50 grams to the balancing mass, m_{25} from Step 1, to get m_{25d}. **Record** m_{25d}, the mass of the cart m_c, and the data used to get the angle of the ramp at 25°

5-1. **Measure and record** the distance from the starting yellow line near the bottom of the ramp to the finishing yellow line near the top of the ramp. Call this distance *s* (*s* should be ~80 cm).

5-2. **Measure and record the time** it takes the cart to go from rest at the starting line up to the finishing line. **Repeat** at least two more times, measuring and recording the time for each run.

5-3. **Calculate** and **record** the average travel time. Call the average time t_1. Use this average in the following calculations.

5-4. **Estimate** and **record** how much uncertainty there is in the time measurement, δt_1. [Note: the experimental uncertainty in the distance is here incorporated as part of the uncertainty in the time.]

5-5. **Calculate** the theoretical acceleration of the cart using **Eq. (6).**

5-6. Using this theoretical acceleration (which is constant), the distance (s) and the initial speed (which should be zero), **calculate** the theoretical time that should have resulted, t_{th1}, using **Eq. (8a).**

5-7. **See** if this theoretical time, t_{th1}, falls within the range of the measured time: $t_1 \pm \delta t_1$.

6. Keep the angle at 25°. **Add** the external mass (the black block) to the cart ($m_{c+} = m_c + m_{block}$) and repeat Step 5, i.e. **add** the same amount of extra mass (30 to 40 grams) to the balancing mass, m_{25+}, to get m_{25+d} and **record** this m_{25+d}, m_{c+}, and the ramp angle of 25°, as you did in Step 5.

6-1-7. Repeat steps 5-1 thru 5-7, relabeling t_1 as t_2, δt_1 *as* δt_2, *and* t_{th1} as t_{th2}.

6-8. ***Is t_2 using m_{c+} more, the same, or less than t_1 using m_c? Can you explain this result?***

REPORT:

1. Answer all the questions posed in the Procedure steps.
2. ***Do the experimental results agree with the theoretical predictions within the range allowed by experimental uncertainty?***
3. ***Are there other experimental uncertainties that we did not explicitly take into account? What are they?***

Note: A table is an excellent way of reporting your data. A table is also an excellent way of reporting your results and making easy comparisons. In the results table list your results for each of the six parts along with the range of uncertainties in your results, and also list the theoretical predictions in this table for easy comparisons.