

HOOKE'S LAW AND POTENTIAL ENERGY

OBJECTIVE: To become acquainted with Hooke's Law for springs and to see how we can describe the behavior of a spring in terms of the energy stored in a spring (potential energy).

Part 1: Hooke's Law and Springs

THEORY:

In this part of the experiment we will test whether a spring obeys Hooke's Law. According to Hooke's Law,

$$F = k|x - x_0|, \quad (1)$$

where F is the size of the force necessary to cause a distortion (stretch the spring in this case), k is the elastic (or spring) constant, x_0 is the position of the end of the spring when it is unstretched (the equilibrium position, shown in Fig. 1A on page 3), and x is the position of the end of the spring when it is stretched or compressed. Note that $|x - x_0|$ is the measure of the distortion (the distance the spring is stretched in this case). If the spring does obey Hooke's Law, we will determine the elastic (or spring) constant, k .

PROCEDURE: Note on units: use either **MKS** units with **Newtons** for force, or **CGS** units with **dynes** for force. A spring is mounted such that it hangs vertically from a support above the table. Weights will be attached to this spring. A ruler is attached next to the spring such that the position of the lower end of the spring can be measured.

Record the position, x , of the lower end of the spring for different applied masses starting at 0 grams, then 100 grams, then increasing the mass by 100 grams each time up to 1,000 grams. Note that in equilibrium the spring force, F , balances the weight, W ($W = mg$) so that $F = W$.

REPORT:

1. **Plot the force, F , vs. the distance, x .** Do you get a straight line? If you do, this says that the force is proportional to the distance ($F \propto [x - x_0]$) and you have verified Hooke's Law, **Eq. (1)**, for the spring.
2. Determine a best straight line through your data points. [Note: if your first two points do not fit the line, then disregard them. We are not taking the weight of the spring itself into account and so Hooke's Law may not seem to hold for very light masses.] **From the slope of the line, determine k , the spring constant,** and record the result. Be sure to **include units**.
3. Determine the x -intercept of the line, i.e. find the value of x for $F = 0$. This value of x should be regarded as a better value for x_0 than the position of the spring without any weight on it. Use this value for x_0 in the second part of this experiment.

Part 2: Energy Stored in a Spring (Potential Energy)

THEORY:

In this part of the experiment we will test the equation for the energy stored in a spring.

Let's take "down" to be the positive direction so that the force exerted on the hanging mass can be written as $F_x = -k(x-x_0)$. Note that if the spring is stretched, x is greater than x_0 and the force is negative, meaning it points upwards. Conversely, if the spring is compressed, x is less than x_0 and the force is positive and points downwards. The work done by the spring force over some small displacement Δx is given by $W = F_x \Delta x$. Recall that the work done by a conservative force, such as a spring force, is the difference between the initial value of the potential energy function and the final value, $W = PE_i - PE_f$. Since ' Δ ' always means final value minus initial value, the work can be expressed as $W = -\Delta PE$. Thus, we can write $\Delta PE = -F_x \Delta x$. We let the displacement get small, so this becomes $d(PE)/dx = -F_x$. Integrating both sides with respect to x gives the potential energy function to within an integration constant:

$$PE = -\int F_x dx = -\int -k(x-x_0) dx = \frac{1}{2}k(x-x_0)^2 + C.$$

We choose $C = 0$ so that PE is zero at $x = x_0$ and the potential energy function is

$$PE_{\text{spring}} = \frac{1}{2}k(x-x_0)^2 \quad (2)$$

Often the distance axis is made so that the equilibrium position is zero ($x_0 = 0$) which simplifies Eq. (2) to $PE = (1/2)kx^2$. However, in our experiment, x_0 is not necessarily zero.

We will test Eq. (2) by setting the spring oscillating and using the Law of Conservation of Energy which states that the energy of a system at a certain time plus any work done **on** the system is equal to the energy of the system at a later time plus any work done **by** the system (which may include frictional terms):

$$KE_i + PE_i + W_{\text{on}} = KE_f + PE_f + W_{\text{by}}. \quad (3)$$

In this experiment we will neglect frictional terms and we will not do any work on the system between the two times we measure the energy. The energy of the system consists of the potential energy stored in the spring [Eq. (2)], the kinetic energy of the oscillating mass [$KE = \frac{1}{2}mv^2$], and the gravitational potential energy of the hanging mass [$PE_{\text{gravity}} = mgh$, where g is the acceleration due to gravity = 9.8 m/s^2 , and h is the height of the mass above some reference point].

In practice, it is hard to measure v , the speed of the mass. We can overcome this by looking for points at which the speed is zero. This happens at $x = x_{\text{bottom}}$ and $x = x_{\text{top}}$. (See Figure 1B.) Hence, the kinetic energy is zero at both places making the conservation of energy equation easier to work with:

$$[PE_{\text{spring}} + PE_{\text{gravity}}]_{\text{at } x_{\text{bottom}}} = [PE_{\text{spring}} + PE_{\text{gravity}}]_{\text{at } x_{\text{top}}} \quad (4)$$

or

$$\frac{1}{2}k(x_{\text{bottom}} - x_0)^2 + mgh_{\text{bottom}} = \frac{1}{2}k(x_{\text{top}} - x_0)^2 + mgh_{\text{top}} \quad (5)$$

We can re-write this last equation as:

$$\frac{1}{2}k(x_{bottom} - x_0)^2 - \frac{1}{2}k(x_{top} - x_0)^2 = mgh_{top} - mgh_{bottom} \quad (6)$$

Now note that $h_{top} - h_{bottom} = x_{bottom} - x_{top} = \Delta x$. From this our energy equation becomes:

$$\frac{1}{2}k [(x_{bottom} - x_0)^2 - (x_{top} - x_0)^2] = mg\Delta x \quad (7)$$

[Note that x_0 is the position when there is no weight stretching the string!]

PROCEDURE: Note on units: use either **MKS** units with **Joules** for energy, or **CGS** units with **ergs** for energy

1. To start this part of the experiment, hang a mass of 1,500 grams from the spring and note the position, x , just as you did in Part 1. Now pull the mass down 5.0 to 10 cm and **record** this position and label it x_{bottom} . Now release the mass and estimate the highest position that the mass reaches before it starts to fall down again. Do this several times and take the average of all the trials. Label this average x_{top} . [For those with phones that can take slow motion, you can use this feature to try to get a more accurate x_{top} position.]

2. Now substitute your values for k (from Part 1), m , g , x_0 , x_{top} , x_{bottom} , and Δx into **Eq. (7)**. Does this equation balance? If it does, within experimental uncertainty, we have verified **Eq. (2)**.

3. Repeat this procedure (Steps 1 and 2) using a 2,000 gram mass.

4. Repeat this procedure (Steps 1 & 2) using a 1,000 gram mass.

REPORT:

1. Perform the calculations described in the Procedure.

2. Does the weight of the mass seem to affect the degree to which **Eq. (7)** balances? Why or why not?

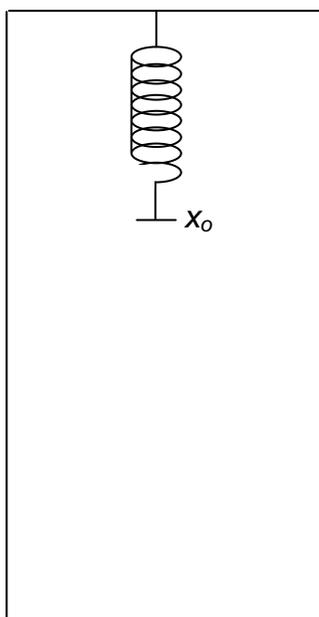


FIGURE 1A

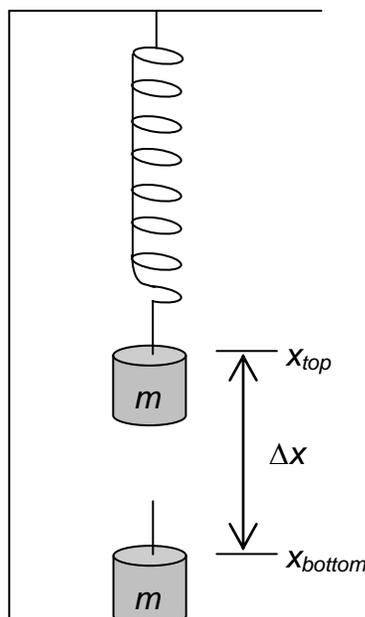


FIGURE 1B