CENTRIPETAL FORCE

OBJECTIVE: To investigate centripetal force and verify Newton's Second Law.

THEORY: A body moving with constant speed in a circular path is constrained to its path by a resultant force constant in magnitude and directed toward the center of the circle (See Fig. 1). This centrally directed net force acting on the body is called **centripetal** force. It provides the centripetal acceleration which keeps the body moving in a circle according to $F_{centripetal} = ma_{centripetal}$. In this experiment, it is supplied by a spring attached to the body (note that the weight of m and the contact force with the cage cancel each other out).

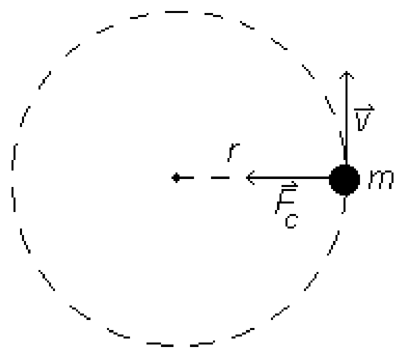

FIGURE 1

According to Newton's Third Law, the body will react to this force from the spring by exerting an equal and opposite force on the spring. Sometimes if the acceleration of the body is not realized (i.e. you are not aware of the circular motion), the $ma_{centripetal}$ term is mistakenly called the ***centrifugal*** force (a "pseudoforce"). When the $ma_{centripetal}$ term is carried to the other side of the Newton's Second Law equation, $\sum \vec{F} = m\vec{a}$, its sign changes and it appears to be a force in the opposite direction from the real centripetal force. This ***apparent*** **force** directed out from the center of the circle is termed the centrifugal force. In this case, the so-called centrifugal force is not really a force, just a misplaced centripetal force term of size $ma_{centripetal}$. (Note that the centripetal force is equal in magnitude but opposite in direction from the so-called centrifugal force.)

The magnitude of the centripetal force is related to the mass and speed of the moving body and the radius of the circular path (measured to the center of gravity of the body):

$$\Sigma F = F_{centripetal} = ma_{centripetal} \quad \text{(Newton's second law)} \qquad (1)$$

where for circular motion:

$$a_{centripetal} = \omega^2 r,\ \ v = \omega r,\ \ \omega = 2\pi f,\ \text{ and } T = 1/f \ \text{ (circular motion eqs.)}$$

or

$$a_{centripetal} = (2\pi f)^2 r = 4\pi^2 f^2 r \qquad (2)$$

where *f* is the frequency (number of revolutions per time) and *r* is the radius of the circle (See Fig. 1). Therefore:

$$\mathbf{F_{centripetal} = ma_{centripetal} = m\,4\,\pi^2 f^2 r} \qquad \mathbf{(3)}$$

All of these terms ($F_{centripetal}$, *f, r,* and *m*) are measurable. In our case, the $F_{centripetal}$ will be supplied by the spring inside the cage.

Part 1. Testing Newton's Second Law: $\sum \vec{F} = m\vec{a}$

PROCEDURE 1:

1) **Record** the mass of the cylinder in the cage, *m*, which is stamped on its flat side facing out. The cage is there to protect us from the mass flying out when it is going around fast, so we don't ask you to take the mass out of the cage to measure it for yourself.

2) See how the cage with the mass, m, enclosed is gripped by the motor. Now **attach** the cage securely to the motor. Tighten the gripper with the key attached to the cord.

3) You should be able to see a pointer on the side of the cage. Turn the motor on, and slowly increase the speed. You should still be able to see the pointer even with the cage rotating. It is easier to see if you get your head at about the same height as the case and look horizontally at the cage. Increase the motor speed until the pointer goes above the central button. Slowly decrease the speed until the pointer goes below the button. Push the frequency button on the apparatus so that the display reads the frequency in rev/min (rpm). **Record** the frequency for which the pointer is definitely up, f_{up}, and for which it is definitely down, f_{down}. This will give a frequency range with the “best” value half way in that range, and the frequency range will give a measure for the uncertainty in the frequency measurement, $f \pm \delta f$. The δf should be less than 10 rpm, that is, the difference between the up frequency and down frequency values should be less than 20 rpm. Turn the motor speed down and then turn the motor off.

4) Next we will determine the force of the spring, which we identify as the $F_{centripetal}$. Remove the cage, and hang it by a piece of string from the stand. The spring should be near the top with the cylindrical mass near the bottom. Hang weights on the low end string to stretch the spring until the pointer is again opposite the button. Record the largest mass, M_{down}, that will not have the pointer move away from the side of the cage; and then record the smallest mass, M_{up}, in which the pointer is definitely away from the side of the cage. Here $M_{up} > M_{down}$. This will give a mass range for M with the “best” value half way in that range, and the mass range will give a measure for the uncertainty in the mass measurement, $M \pm \delta M$. The δM should be less than 100 grams, that is, the difference between the up mass and down mass values should be less than 200 grams. The spring force now equals the weight of all the mass attached to the end of the spring [$F_{spring} = (M+m)g$].

5) While the cage is hanging with either M_{up} or M_{down} on it, measure the radius, *r,* from the axle (center white line) to the line on the body (which marks the center of gravity). Be sure to estimate the uncertainty δr in this measurement.

REPORT: Now we are in a position to **check Newton's Second Law**.

1. Since we now know a range for *f* and a value for *r,* we can **calculate a range of values for $a_{centripetal}$** using f_{up} and f_{down} in our equation for acceleration: $a_{cengtripetal} = 4\pi^2 f^2 r$. Do so. Watch your units: the frequency from the motor is in rpm (rev/min), and needs to be converted into cycles/second. Use either MKS units (meters, kilograms, seconds with force in Nt), or CGS units (centimeters, grams, seconds with force in dynes). Don’t mix MKS with CGS units.

2. The tension in the spring [which is equal to $(M+m)g$ from step 4 in the above procedure] provides the centripetal force $F_{centripetal}$. **Calculate** a **range of values for $F_{centripetal}$**, using M_{up}+m and M_{down}+m times g. Recall that weight is a force and is equal to mass times g.

3. We now have a value for m and ranges of values $a_{centripetal}$ and $F_{centripetal}$. **Does the range of values for $F_{centripetal}$ *overlap the range of values for* $ma_{centripetal}$?**

4. If the two ranges overlap, then we have verified Newton’s 2nd Law as well as we can. If the two ranges don’t overlap, are there other uncertainties besides those in f and M that can account for the fact that the ranges for F and ma don’t overlap?

PROCEDURE 2:
Now readjust the tension in the spring by turning the dial by the spring from the zero setting to the 20 setting (or vise-versa), and repeat the experiment with the spring set at this new tension.

After you have completed the Report section, answer the questions below.

Part 2. Questions About Circular Motion

Answer the following questions using the quantities obtained in Procedure 1.
Answer in complete sentences or equations, such as:

$W = m\,g$ = (value for mass in kg) x (9.8 m/sec^2) = (value for weight in Nt).

1. What is the weight (W) of the revolving body of mass (m)?

2. When the mass (m) is being spun around, what is the centripetal force ($F_{centripetal}$) on it (magnitude and direction)?

3. What causes the centripetal force, that is, what physical body or agent exerts the force necessary to keep the object moving in a circle?

4. What causes the spring to be stretched? (HINT: consider Newton's Third Law)

5. From the point of view of an observer riding on the revolving frame, there is no apparent motion and hence no apparent acceleration. However, the observer will see that the spring is stretched! To this observer, this mass behaves exactly as if it were in an **effective gravitational field** with an acceleration due to gravity (g_{eff}). Recall that the revolving mass stretched the spring in our experiment exactly like a heavy weight did. Therefore the observer would see the mass pulling "down" on the spring with a weight (W_{eff}) that is equal in size but opposite in direction to the centripetal force that we observe in the laboratory frame of reference:

$$W_{eff} = mg_{eff} = -F_{centripetal} = ma_{centripetal}$$

From this point of view, what is the effective weight (W_{eff}) of the revolving mass (magnitude and direction)?

6. What is the value of this g_{eff} (magnitude and direction)?

7. Using the same radius as you used above, what frequency (f) expressed in rps and rpm would the mechanism have to rotate at to make the effective weight equal to 100 times the stationary weight , i.e., W_{eff} =(100)W ?

8. Suppose the spring broke while the mass was being rotated. Also assume that the mass flies off and is not immediately stopped by the end of the rotating mechanism, and neglect the effect of the real gravity in the room. What path would the mass appear to take as seen by an observer in the room?

9. What path would the mass in (8) above appear to take as seen by the observer located on the revolving frame? At first, confine your analysis to the time immediately following the break. Then extend your analysis to later times.

10. a) Draw a diagram showing the positions of the end of the rotating bar and the flying mass as seen from the lab frame's point of view at the following five times:
t_1: the instant the mass starts to leave the end of the rotating bar,
t_2: the instant when the bar has rotated 90° from the release point,
t_3: the instant when the bar has rotated 180° from the release point,
t_4: the instant when the bar has rotated 270° from the release point,
t_5: the instant when the bar has rotated a full 360° from the release point.

b) Draw a diagram showing the position of the mass relative to the end of the bar as seen from the point of view of an observer riding on the end of the rotating bar for each of the five times in part (a) above.

11. Finally, with your instructor, run the computer program on centripetal motion and compare your sketches of the motion (question 10) with those generated by the computer, and see if your answers to question 10 are correct. If the computer is off, turn it and the monitor on. From the desktop, click on the CentForce icon to run the centripetal force routine.

This idea of effective gravity is the principle that explains the operation of a “centrifuge”. The term "centrifugal force" is sometimes given to what we have called the effective gravity. Hence the name centrifuge. Note that centrifugal force is not a real force and does not belong in the $\sum \vec{F}$ term of $\sum \vec{F} = m\vec{a}$. However, to an observer on the rotating frame it appears that centrifugal force is real. Such an observer does not measure the same acceleration as the stationary observer measures and so must *invent* a force, the fictitious "centrifugal force", to account for the difference in acceleration.

Since the earth is spinning around its axis, do we see any of signs of centripetal acceleration (or “centrifugal” forces)? If you follow the weather, you know that winds go around low and high pressure regions rather than into or out of these regions. This is due to the fact the earth is rotating, and we can account for this “strange” behavior by taking into account the centripetal acceleration of the air.