THE ATWOOD MACHINE
(Newton's Second Law and the Conservation of Energy)

OBJECTIVE: To study the relation of masses and accelerations.

METHOD: Consider the Atwood machine shown in Fig. 1. A pulley is mounted on a support a certain distance above the floor. A string with loops on both ends is threaded through the pulley, and different masses are hung from both ends. The smaller mass is placed near the floor and the larger mass near the pulley (the pulley can be adjusted to the appropriate height). The masses are then released and the time for them to exchange places is measured.

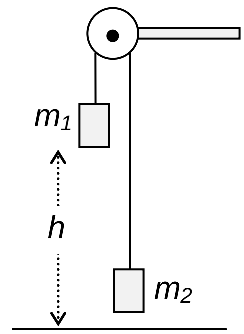

FIGURE 1

As always be careful of units. Use either MKS or CGS, but don't mix them.
For MKS, units of force are in Newtons; units of energy are in Joules.
For CGS, units of force are in dynes; units of energy are in ergs.

Part 1. Newton's Second Law

THEORY: Consider the larger mass, m_1. There are two forces acting on it. One is the force of gravity, $W_1 = m_1 g$, pulling it downward. The other force is the tension in the string, T_1, which is pulling it upward. Taking up to be the positive direction, Newton's 2nd Law gives

$$\Sigma F_y = -m_1 g + T_1 = m_1 a_1 \, . \qquad (1)$$

Now consider the smaller mass, m_2. Again there are two forces acting on it. One is the force of gravity pulling it downward. The other force is the tension in the string pulling it upward. Thus, Newton's 2nd Law gives

$$\Sigma F_y = -m_2 g + T_2 = m_2 a_2 \qquad (2)$$

Because the string is attached to both masses, $-a_2 = a_1 = a$. We now assume that the string's mass is much less than either of the hanging masses and that the pulley does not take any energy as the masses move. This allows us to say that $T_1 = T_2 = T$. (In reality, the pulley does take energy to rotate and the tensions won't be equal if it rotates. However, we assume that the pulley's motion takes very little energy from the system so we can approximate it as being stationary. Another way to put this approximation is that we are using a "massless" pulley and a "massless" string.) With this assumption, we can write Eq. (2) as

$$T = m_2 (-a) + m_2 g \; . \qquad (3)$$

Substituting T from Eq. (3) into Eq. (1) gives

$$-m_1 g + (-m_2 a + m_2 g) = m_1 a$$

or

$$a_{\text{theory}} = g(-m_1 + m_2)/(m_1 + m_2) \; . \qquad (4)$$

Since $m_1 > m_2$, a_{theory} will be negative – which means down. We will ignore the minus sign since we are only interested in the magnitude, so

$$a_{theory} = g(m_1 - m_2)/(m_1 + m_2) \; . \qquad (4t)$$

By the appropriate choice of masses we can choose any acceleration we wish up to the value of *g* itself. What choice of masses would give $a = 0$? What choice would give $a = g$?

PROCEDURE:

1) Measure all of the six masses and record their values. In the calculations below, **use your measured values for the m's**, not the values stamped on the masses.

2) Take the masses labeled $m_1 = 50$ gm and $m_2 = 40$ gm. Allow m_1 to fall a distance h from near the pulley to the floor. **Measure** the distance of the fall (make $h = 100$ cm by adjusting the pulley's position) and **measure** the time of the fall. Repeat at least **three times** and take an average.

3) Recall the equations of motion for constant acceleration in one-dimension,

$$x = x_o + v_o t + \tfrac{1}{2} a t^2 \qquad (5)$$

and

$$v = v_o + a t. \qquad (6)$$

In our case, x = 1 m, $x_o = 0$ m, and $v_o = 0$ m/s, so Eq. (5) becomes

$$\mathbf{a_{experimental} = 2*(1\ m) / t_{avg}^2} \qquad (5e)$$

and

$$\mathbf{v_{experimental} = a_{experimental} * t_{avg}} \qquad (6e).$$

(a) Using Eq. (**5e**), determine the experimental acceleration using your average experimental time and compare it to the value of a_{theory} predicted by Eq. (**4t**).

(b) Are the theoretical and experimental accelerations the same? If not, is the **uncertainty** in the time measurement (usually 0.1 sec) sufficient to account for this difference? To answer this, use Eq. (5e) to calculate the experimental acceleration, $\mathbf{a^-}$ using $\boldsymbol{t^- = (t_{avg} - 0.1\ sec)}$, and then calculate the experimental acceleration, $\mathbf{a^+}$ using $\boldsymbol{t^+ = (t_{avg} + 0.1\ sec)}$. Does the theoretical acceleration fall between these two values? If the theoretical acceleration does not fall in this range, what else could account for this disagreement between theory and experiment?

4) **Repeat** Steps 1&2 for a machine with the pair of masses labeled $m_1 = 100$ gm & $m_2 = 90$ gm. Repeat for a machine with the pair of masses labeled $m_1 = 200$ gm and $m_2 = 190$ gm.

REPORT: Be sure to include all the data (what you measured) including **each** of your times. Perform the calculations and answer the questions raised in the Procedure.

Report your results with a line for each of the three machines such as

m_1 (kg)	m_2 (kg)	t_{avg} (sec)	t^+ (sec)	t^- (sec)	a_{exp} (m/s^2)	a^+ (m/s^2)	a^- (m/s^2)	a_{theory} (m/s^2)

Part 2. Conservation of Energy

THEORY: We consider the system that is conserving energy as consisting of both masses. Again, we neglect the masses of the pulley and the string and assume no energy is lost to drag. The Law of Conservation of Energy states

$$KE_{1i} + KE_{2i} + PE_{1i} + PE_{2i} + W_{T1} + W_{T2} = KE_{1f} + KE_{2f} + PE_{1f} + PE_{2f}. \quad (7)$$

Here, KE_1, KE_2, PE_1, PE_2 are the kinetic and potential energies of mass 1 and mass 2, W_{T1} is the work done by tension on mass 1, and W_{T2} is the work done by tension on mass 2. Since the tensions are equal with our assumptions, $W_{T1} + W_{T2} = 0$. (Why?)

Recall that kinetic energy is the energy of motion and that gravitational potential energy is the energy due to the height of an object. Specifically, $KE = ½mv^2$ and $PE = mgh$. The masses start at rest ($v_i = 0$) and end up traveling at the same speed ($v_f = v$). Let's call the floor height zero and the final height of the smaller mass h = 1 m. You should see that we then can write Eq. (7) as

$$m_1gh = ½(m_1 + m_2)v^2 + m_2gh. \quad (8)$$

We can solve Eq. (8) for the theoretical velocity (assuming no E_{lost}):

$$\mathbf{v_{theory} = [2(m_1 - m_2)gh / (m_1 + m_2)]^{1/2}} \quad (\mathbf{8t})$$

PROCEDURE:

1) (a) From the Law of Conservation of Energy, Eq. (**8t**), determine what the **theoretical** final speed should be for both masses..

(b) Now use the experimentally found time, t_{avg}, and the experimental value of the acceleration, a_{exp}, in the equation for constant acceleration, Eq. (**6e**), to find the **experimental** final speed, v_{exp}, of the masses.

(c) Are the theoretical and experimental final speeds the same? If not, can you explain why not?

(d) Find the **range** for the experimental v using Eq. ($6e^{+/-}$):

$$v^+ = a^+ * t^+ \quad \text{and} \quad v^- = a^- * t^- . \quad (6e^{+/-})$$

2) If the theoretical and experimental accelerations do not agree within experimental uncertainty in the time measurement, and if theoretical and experimental speeds do not agree within experimental uncertainty in the time measurement, there may have been a sizeable amount of energy leaving the system. Recall that our theoretical system consists of only the two weights. Calculate how much energy was lost by the system. This can be done by subtracting the right side of Eq. (8) from the left side, i.e.

$$\mathbf{E_{lost} = m_1gh - ½(m_1 + m_2)v_{exp}^2 - m_2gh} . \quad (\mathbf{9})$$

When you do this calculation, be sure to use your experimental speed for v. You should obtain a **range** of possible energy losses rather than a single value because of the uncertainty in your speed. To get this range, calculate E_{lost}^+ using v^+ and E_{lost}^- using v^-.

REPORT:

1. Perform the calculations and answer the questions raised in the Procedure for EACH of the three machines. Be sure to show sample calculations for each type and include all units in your sample calculations.

2. In your report **use tables** to display your results with a line for each of the three machines - see previous above for a first table for the Newton's 2nd Law part, and below for a second table for the Conservation of Energy part:

m_1	m_2	v_{exp}	v^{+}	v^{-}	v_{theory}	$E_{lost\text{-}exp}$	E_{lost}^{+}	E_{lost}^{-}
(kg)	(kg)	(m/s)	(m/s)	(m/s)	(m/s)	(J)	(J)	(J)

3. List the places that any lost energy goes.

4. Does it seem that the amount of energy loss you calculated for the three sets of masses depends on the weight hung over the pulley? Justify your answer. HINT: friction with the pulley should increase with the heavier masses; air resistance should increase with the faster masses.

5. As always, include a discussion of experimental uncertainties and assumptions.