ACCELERATION DUE TO GRAVITY

OBJECTIVE: To study uniformly accelerated linear motion and to determine the acceleration due to gravity, *g*.

THEORY: Velocity is defined as the rate of change of position, and acceleration is defined as the rate of change of velocity. For linear motion, then

$$v \equiv \Delta s / \Delta t \qquad a \equiv \Delta v / \Delta t \qquad (1,2)$$

If *a* is CONSTANT, we can begin with these equations and derive equations for the velocity,v, and displacement, s, of an object after the elapse of any length of time, *t*:

$$v = v_o + at \qquad (3)$$

$$s = s_o + v_o t + \tfrac{1}{2}at^2. \qquad (4)$$

The best example of constant acceleration is that provided by gravity in the absence of air resistance. We can ignore the resistance of air if the body is small and travels only a short distance. (Actually, for large distances, this resisting force is significant and eventually results in motion with a constant velocity instead of constant acceleration. Also, the value of *g* varies somewhat according to height and geographical location.)

Experimentally we will not be able to get values of instantaneous velocity. Instead we use very small intervals of time, and calculate the values of AVERAGE velocities over those intervals. As an interval gets smaller, the average value approaches the instantaneous value at the midpoint of the interval:

$$v_{avg} = \Delta s / \Delta t = (s_2 - s_1) / (t_2 - t_1) \approx v_{1.5} \ . \qquad (5)$$

And the average acceleration is given by:

$$a_{avg} = \Delta v / \Delta t = (v_{2.5} - v_{1.5}) / (t_{2.5} - t_{1.5}) \approx a_2 \ . \qquad (6)$$

PROCEDURE:

1. Connect the synchronous timer to the two posts on the board attached to the desk (this is usually already set up for you) Turn on the timer (left switch) to allow it to warm up. The high voltage is not on until the button on the remote switch is pressed. Do not touch the wires or terminals when this switch is pressed.

2. Cut off a piece of tape about one meter long. Feed one end of the tape between the two wire points in the apparatus, and attach the weighted clip to the lower end of the tape.

3. One partner should stand on a ladder to hold the upper end of the tape directly above the apparatus while the other turns on the high voltage by pressing the button. As soon as the high voltage is turned on, the person on the stool should hear this and drop the tape. The person holding the switch should keep the switch pressed until the tape falls completely through the

apparatus. This will leave a trace of the falling object's path by means of dots on the tape. There are two set-ups. Groups will take turns.

4. Starting with a dot that is several centimeters from the first dot, mark **every other** dot by circling it. Label these dots 1, 2, 3, etc. While the dots are spaced at intervals of 1/60 second, calculations will be made on the basis of 1/30 second by using every other dot. You can use the table on the last page to record your measurements.

5. Lay the tape out flat and place the meter stick edgewise on the tape. Why? Do not place the end of the meter stick on the first dot, but use some position such as the 10 cm mark. Why? Record the positions and times in the table on the last page under the Raw Data heading. Note that we are recording these positions as positive values, but the actual motion was going down. This means that **down is positive** in our case. This choice of down as positive is not normal, but it is allowed and is certainly more convenient than putting minus signs in for all the positions.

6. Determine Δs, the distance between each adjacent pair of circled dots. Note that Δt, the time between dots, is always 1/30 sec. Record these values in the table under the Calculated Data for Velocity heading **between** every two consecutive numbered dots.

7. Calculate the average velocity during each interval according to **Eq. (5)**. [HINT since Δt = 1/30 sec, dividing by (1/30)sec is the same as multiplying by 30/sec.] Note that the average velocity is equal to the instantaneous velocity at the MIDPOINT in time for that interval if the acceleration is constant. Indicate this by recording this velocity **between** dot numbers just as you did for Δs and Δt above. Also, calculate the midpoints of the time intervals and record these in the time column next to the average velocity values. You will plot the average velocities versus these times in Graph 1.

8. Find the changes in velocity, Δv, from one velocity to the next and record these **between** the v values under the Calculated Data for Acceleration heading. Note that Δt, the time between two consecutive average velocity times, is again always 1/30 sec. Record the Δt values in the column to the right of the Δv values.

9. From the Δv and Δt values, calculate the values of the acceleration using **Eq.(6)** and record them. [Again, dividing by (1/30)sec is the same as multiplying by 30/sec.]

10. Take the average of all the accelerations. This is our **first way of determining the acceleration** due to gravity.

11. Using 980 cm/sec^2 as the correct value for g, determine the percent error.

12. Look over your experimental procedure and determine the major sources of uncertainty that might explain your error.

GRAPH #1: *v* versus *t*

1. **Graph** (using a spreadsheet with a Scatter type graph) ***v* vs *t*** using your data from the Data Table. Be sure to label your axes and include units – do this inside a spreadsheet program if possible. (**Be careful**. The *v*'s you have calculated are for the times *between* dots, not the times at the dots! Your data and graph should reflect this fact.)

2. Now have the spreadsheet program draw the best straight line through these points. For constant acceleration we know fro.m **Eq. 3**: $v = v_o + at$. Thus, the slope of the *v* vs *t* line should be the acceleration. **Find the slope** of this line – you can do this by having the spreadsheet program print out the equation for the best fit line (in the math form of **y = mx + b**). Now put this equation into physics form by identifying **y** as the velocity, **v**; **x** as the time, **t**; **m** as the acceleration, **a**; and **b** as the v-intercept, v_o . Remember, the slope has UNITS, and those units should be those of acceleration! **This is a second way of determining the acceleration** due to gravity. Record this value of g in your results and compare to the accepted value of g and to the average value of g obtained in the table.

3. **Determine** v_o from your graph (v_o is the *v*-intercept which is the value of v at t = 0).

4. Finally, determine the time at which the motion actually started, that is, **determine** t_o**,** the time when *v* = 0. If your first dot was recorded as being at t=0, this should be a negative time since we started recording positions AFTER we let the weight drop.

GRAPH #2: *s* versus *t*

1. **Graph *s* vs. *t*** using your raw data using the Scatter type of graph. According to **Eq. (4)** *s* is a function of *t*, but not a linear one. Thus, this graph should not be a straight line. According to **Eq. (4)**, this second graph should be a **parabola**. Does it look like it might be?

2. Now have the spreadsheet program draw the best polynomial of order 2 curve (a parabola) through these points, and have the program write the math equation on the graph. This equation will be in the "math" form, $y = ax^2 + bx + c$. Rewrite this equation in physics form ($s = s_o + v_o t + \frac{1}{2}at^2$, **Eq. 4**), by identifying **y** as the position, **s**; and **x** as the time, **t.** Be sure to include units for the math constants **a**, **b**, and **c**. By considering theory: $s = s_o + v_o t + \frac{1}{2} at^2$, **determine the acceleration due to gravity**. You can do this by noting that the coefficient of the t^2 term is ½**a** in the physics form and in the math form it is **a** (so the acceleration **a** = 2**a**). This gives us a **third value for the acceleration** due to gravity, **g**. Record it in your results, and compare it to the accepted value of g and to the previous two values of g determined earlier.

3. The **slope** of this curve at any point is *ds*/*dt*, which we recognize as the velocity at that time. Choose one of the recorded points on the s vs t curve, and **draw a tangent to the curve** at that point. (Remember that a tangent line touches the curve at one point but does not cross the line at that point.) Determine the slope of this tangent line (which is also the slope of the curve at that point) by choosing two points on the tangent line. Compare this slope value (with units) with the calculated velocity at that tangent point from the table. Again, be careful - your calculated velocities are between points. Record this comparison in your results.

REPORT:

1. Include the data table and your two graphs.
2. Comment on what your graphs say (answer the questions in each of the parts above). Do they agree with the theory?
3. Record your three values for g and compare them to one another and to the accepted value of g. (A table is a good way of displaying these results.)
4. Record the value of the slope of your tangent line, and show the comparison to the value of the velocity at that tangent point from your data table.
5. Finally, include an analysis of experimental uncertainty and the resulting error. In particular, consider how your graphs both show and compensate for the uncertainties.

DATA TABLE

	Raw Data		Calculated Values for v				Calculated Values for a		
Dot #	***t*** ( )	***s*** ( )	**Δ*s*** ( )	**Δ*t*** ( )	***v*** ( )	***t*** ( )	**Δ*v*** ( )	**Δ*t*** ( )	***a*** ( )
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									

Average *a* ____________