1. Do p. 782 #24.
\[ \sin(A) = 3/5 \text{ and } A \text{ is in quadrant 1. } \cos(B) = 12/13 \text{ and } B \text{ is in quadrant 4.} \]
The triangle with angle A has an adjacent leg of 4 by the Pythagorean theorem. 
The triangle with angle B has an opposite leg of -3 by the Pythagorean theorem.

a) Find \( \sin(A + B) \). Use the sine of a sum formula.
\[ \sin(A + B) = \sin(A) \cos(B) + \cos(A) \sin(B) = 3/5 \cdot 12/13 + 4/5 \cdot (-3/5) = 16/65 \]

b) Find \( \cos(A - B) \). Use the cosine of a difference formula.
\[ \cos(A - B) = \cos(A) \cos(B) + \sin(A) \sin(B) = 3/5 \cdot 12/13 + 4/5 \cdot (-3/5) = 33/65 \]

c) Find \( \tan(A - B) \). Use the tangent of a difference formula.
\[ \tan(A - B) = \frac{\tan(A) - \tan(B)}{1 + \tan(A) \tan(B)} = \frac{1}{1 + 3} = \frac{1}{4} \]

2. Do p. 783 #34.
Show \( \frac{\cos(A + B)}{\cos(A - B)} = \frac{1 + \tan(A) \tan(B)}{1 - \tan(A) \tan(B)} \)
Start with the right side. Use \( \tan(w) = \frac{\sin(w)}{\cos(w)} \).

right = \frac{1 + \sin(A) \sin(B)}{1 + \cos(A) \cos(B)} \quad \text{Multiply top and bottom by } \cos(A) \cos(B).

right = \frac{\cos(A) \cos(B) - \sin(A) \sin(B)}{\cos(A) \cos(B) + \sin(A) \sin(B)} \quad \text{Use cosine of sum and of difference identities.}

right = \frac{\cos(A + B)}{\cos(A - B)} = \text{left}

3. Do p. 783 #46.
Find \( \cos\left(\frac{\pi}{3}\right) \). Use the half angle identity for cosine with \( \frac{\pi}{6} = \frac{1}{2} \cdot \frac{\pi}{3} \).
\[ \cos\left(\frac{\pi}{3}\right) = \pm \sqrt{\frac{1 + \cos(\frac{\pi}{6})}{2}} \]
Choose plus since \( \frac{\pi}{6} \) is in quadrant 1.

\[ \cos\left(\frac{\pi}{3}\right) = \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 + \sqrt{3}}{4}} \]

Multiply top and bottom inside the outer sqrt by \( \sqrt{2} \).

\[ \cos\left(\frac{\pi}{3}\right) = \sqrt{\frac{2 + \sqrt{2}}{4}} \]

4. Do p. 784 #54.
\[ \sin(\pi/10) = -4/5 \text{ and } t \text{ is in quadrant 3.} \]
The side adjacent to angle t has length -3 by the Pythagorean theorem.

Use the double angle formulas. \( \sin(2t) = 2 \sin(t) \cos(t) = 2 \cdot (-4/5) \cdot (-3/5) = 24/25 \)
\[ \cos(2t) = 1 - 2 \sin(t)^2 = 1 - 2 \cdot (-4/5)^2 = -7/25 \]
\[ \tan(2t) = \frac{\sin(2t)}{\cos(2t)} = -\frac{24}{7} \]

5. Do p. 784 #68.
Show \( 32 \sin(2t) \cos(t)^2 = 2 - \cos(2t) - 2 \cos(4t) + \cos(6t) \)
Start by working with each term of the right side.
\[ \cos(6t) = \cos(4t + 2t) = \cos(4t) \cos(2t) - \sin(4t) \sin(2t) \quad \text{by the cosine of a sum formula.} \]
\[ \cos(6t) = (\cos(2t)^2 - 1) \cos(2t) - 2 \sin(2t) \cos(2t) \sin(2t) \quad \text{by double angle formulas for } 4t. \]
\[ \cos(6t) = 2 \cos(2t)^3 - 3 \cos(2t) - 2 \cos(2t) \sin(2t)^2 \]
\[ \cos(6t) = 2 \cos(2t)^3 - 3 \cos(2t) - 2 \cos(2t) (1 - \cos(2t)^2)^2 \quad \text{since } \sin(w)^2 = 1 - \cos(w)^2 \]
\[ \cos(6t) = 4 \cos(2t)^3 - 3 \cos(2t) \quad \text{by algebra.} \]

Now \( \cos(4t) = 2 \cos(2t)^2 - 1 \quad \text{by double angle formula for } \cos(2t). \)
So \( -4 \cos(4t) = -4 \cos(2t)^2 + 2 \)
Then \( \text{right} = 2 - \cos(2t) - 2 \cos(4t) + \cos(6t) \)
\[ \text{right} = 2 - \cos(2t) - 4 \cos(2t)^2 + 2 + 4 \cos(2t)^3 - 3 \cos(2t) \]
So \( \text{right} = 4 - 4 \cos(2t) - 4 \cos(2t)^2 + 4 \cos(2t)^3 = 4(1 - \cos(2t))(1 - \cos(2t)^2) \)
\[ \text{right} = 4(1 - \cos(2t))(\sin(2t)^2)^2 \quad \text{by a Pythagorean identity.} \]
\[ \text{right} = 4(1 - (1 - 2 \sin(t)^2))(2 \sin(t) \cos(t))^2 \]
\[ \text{right} = 4 \cdot 2 \sin(t)^2 \cdot 4 \sin(t)^2 \cos(t)^2 \]
right = 32 \sin(t)^4 \cos(t)^2 = left

6. Do p. 784 #71.
Show sec(2t) = \frac{\cos(t)}{\cos(t) + \sin(t)} + \frac{\sin(t)}{\cos(t) - \sin(t)}

Add the fractions on the right side. LCD = (\cos(t) + \sin(t))(\cos(t) - \sin(t)).
right = \frac{\cos(t)(\cos(t) - \sin(t)) + \sin(t)(\cos(t) + \sin(t))}{(\cos(t) + \sin(t))(\cos(t) - \sin(t))}
right = \frac{\cos(t)^2 - \cos(t)\sin(t) + \sin(t)\cos(t) + \sin(t)^2}{(\cos(t) + \sin(t))(\cos(t) - \sin(t))}$\frac{1}{(\cos(t) + \sin(t))(\cos(t) - \sin(t))}$

Use the distributive law in the top.
right = \frac{\cos(t)^2 - \cos(t)\sin(t) + \sin(t)\cos(t) + \sin(t)^2}{(\cos(t) + \sin(t))(\cos(t) - \sin(t))}
right = \frac{\cos(t)^2 - \cos(t)\sin(t) + \sin(t)\cos(t) + \sin(t)^2}{(\cos(t) + \sin(t))(\cos(t) - \sin(t))}$\frac{1}{(\cos(t) + \sin(t))(\cos(t) - \sin(t))}$

FOIL in the bottom.
right = \frac{\cos(t)^2 - \cos(t)\sin(t) + \sin(t)\cos(t) + \sin(t)^2}{(\cos(t) + \sin(t))(\cos(t) - \sin(t))}$\frac{1}{(\cos(t) + \sin(t))(\cos(t) - \sin(t))}$

Use a double angle formula for cosine.
right = \frac{\cos(t)^2 - \cos(t)\sin(t) + \sin(t)\cos(t) + \sin(t)^2}{(\cos(t) + \sin(t))(\cos(t) - \sin(t))}$\frac{1}{(\cos(t) + \sin(t))(\cos(t) - \sin(t))}$

= \frac{1}{\cos(2t)} = sec(2t) = left

7. Do p. 809 #3.
y = -2\cos(x). Amplitude is 2. Period is 2\pi. Phase shift is zero. Vertical shift is zero.
Five points to plot are (0, -2), (\pi/2, 0), (\pi, 2), (3\pi/2, 0), (2\pi, -2).

y = \sin(-x - \pi/4) - 2
Use sin(-w) = -\sin(w).
y = -\sin(x + \pi/4) - 2. Amplitude is 1, period is 2\pi. Phase shift = -\pi/4. Vertical shift is -2.
Five points to plot are (\pi/4, -2), (\pi/2 + \pi/4, -3), (\pi/2 + \pi/2, -2),
(\pi + \pi/2, -1), (\pi + \pi/4, -2)

Five points to plot are (\pi/4, -2), (\pi/4, -3), (\pi/2, -2), (\pi/2 + \pi/2, -1), (\pi/2 + \pi/4, -2)
s(x) = \sin(-x - \pi/4) - 2

y = \tan(x - \pi/3). Period is \pi. Phase shift is \pi/3.
Domain is all reals except where x - \pi/3 = \frac{\pi}{2} + k\pi for an integer k.
Domain is all reals except \frac{\pi}{6} + k\pi for an integer k. Range is all reals.
Five points to plot are (\pi/3, 0), (\pi/3 + \pi/2, 1), (\pi/3 + \pi/2, -1), (\pi/3 - \pi/2, -1), (\pi/3 - \pi/2, \sqrt{3})
After arithmetic the points are (\pi/3, 0), (\pi/12, 1), (\pi/12, \sqrt{3}), (\pi/12, -1), (0, -\sqrt{3})
t(x) = \tan(x - \pi/3)
10. Do p. 809 #18.
y = \frac{1}{3} \sec\left(\frac{x}{2} + \frac{\pi}{3}\right)
The period is \frac{2\pi}{\frac{1}{2}} = 4\pi
The phase shift is \frac{-\frac{\pi}{3}}{\frac{1}{2}} = \frac{2\pi}{3}
Domain is all reals except where \cos\left(\frac{x}{2} + \frac{\pi}{3}\right) = 0
\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{2} or x = \frac{\pi}{6}
Domain is all reals except \frac{\pi}{6} + 2k\pi for integer k.

Six point to plot for y = \sec( w ) are 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}
Solve \frac{x}{2} + \frac{\pi}{3} = w for x.
x = 2(w - \frac{\pi}{3})

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<thead>
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<th>x</th>
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<th>y</th>
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<tr>
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<td>\frac{5\pi}{4}</td>
<td>\frac{\sqrt{3}}{2}</td>
</tr>
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</table>

For the secant curve, points to plot are the points ( x, y ) in the above table.

f(x) = \frac{3}{2} \cos(2x) - \frac{3\sqrt{3}}{2} \sin(2x) + 6. Write f as one sine and as one cosine function.
Use # 36 and 35 on p. 810.
Here b = \frac{3}{2} and a = -\frac{3\sqrt{3}}{2}
So a^2 + b^2 = \frac{9}{4} + \frac{27}{4} = 9
Then \sqrt{a^2 + b^2} = 6.
tan(\phi) = b/a = \frac{1}{\sqrt{3}}.
So \phi = \frac{\pi}{6}.
So S(x) = 6\sin(2x + \frac{\pi}{6}) + 6.
By #35 C(x) = 6\cos(2x + \frac{\pi}{3} + \frac{\pi}{2}) + 6 = 6\cos(2x + \frac{\pi}{3}) + 6.