Quadratic Drag Model

```
> restart:with(DEtools):
```

This is a single differential equation for the quadratic drag model using the signum function. Here we have $r = 2$, $\frac{b}{m} = 8$, and $g = 32$

```
> deq:=diff(v(t),t)=9.81-8*v(t)^2*signum(v(t));
```

$$deq := \frac{d}{dt}\, v(t) = 9.81 - 8\, v(t)^2\, \mathrm{signum}(v(t))$$

Let's find the general solution.

```
> dsolve(deq,v(t));
```

$$v(t) = \frac{3}{40}\sqrt{218}\, \tan\left(\frac{3}{5}\sqrt{218}\ \_C1 + \frac{3}{5}\sqrt{218}\ t\right), v(t) = \frac{3}{40}\sqrt{218}\, \tanh\left(\frac{3}{5}\sqrt{218}\ \_C1 + \frac{3}{5}\sqrt{218}\ t\right)$$

We get two solutions, with the second being the appropriate one for our case.

To find the constant solution, we set $\frac{dv}{dt} = 0$ and solve the resulting algebraic equation. We first need to replace each instance of $v(t)$ with v.

```
> de:=subs(v(t)=v,rhs(deq));
```

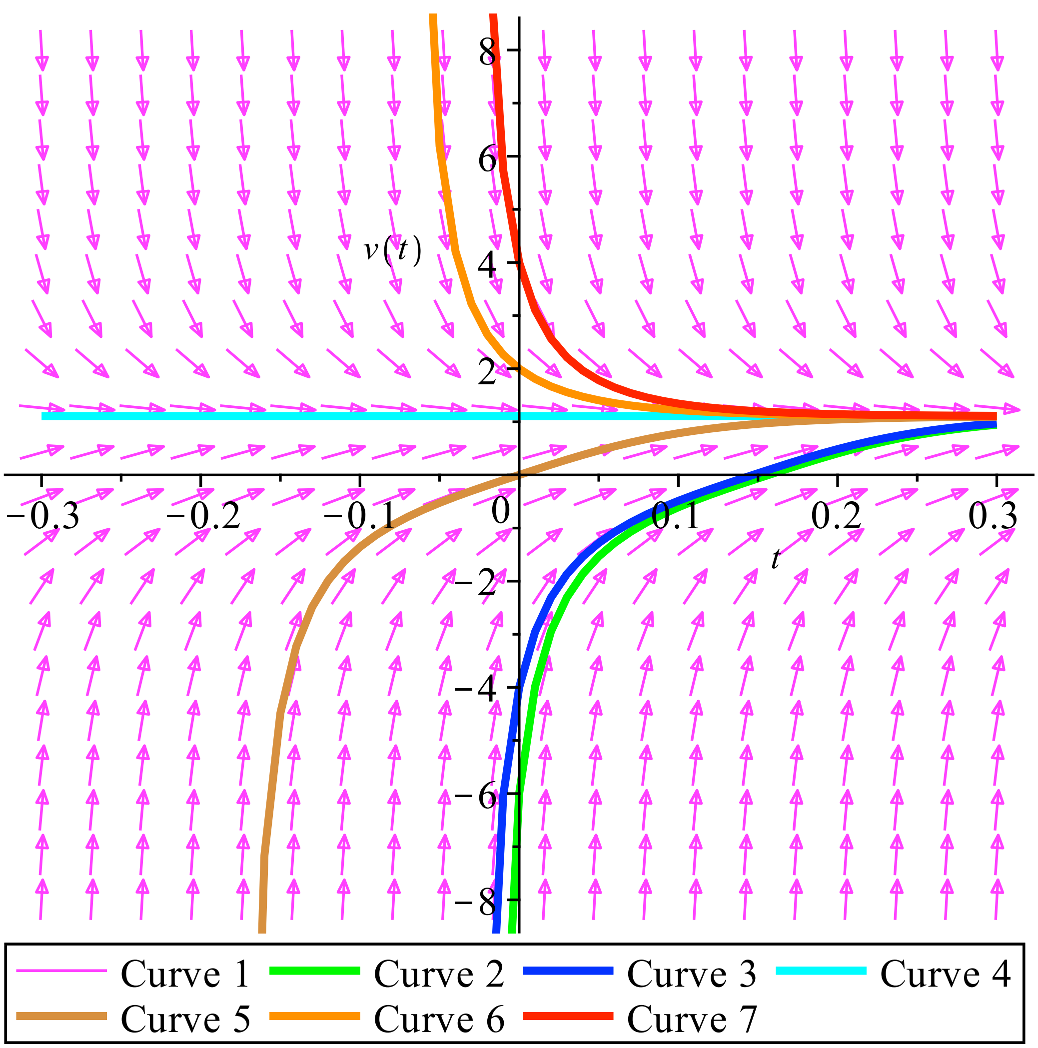
$$de := 9.81 - 8\, v^2\, \mathrm{signum}(v)$$

```
> solve(de=0,v);
```

$$1.107361730$$

Thus $v = -2$ is the constant solution. Finally, let's plot the direction field and several solution curves.

```
> DEplot (deq, v(t), t=-0.3..0.3, v=-8..8,stepsize=.01,[[0,-6],[0,
  -4],[0,1.107361730],[0,0],[0,2],[0,4]],arrows=medium,color=magenta,
  linecolor=[green,blue,cyan,gold,coral,red]);
```


v(t)
t
Curve 1
Curve 2
Curve 3
Curve 4
Curve 5
Curve 6
Curve 7