Orange Juice in a Vat

A vat contains 50,000 gallons of orange juice, 10% of which is concentrate. A mixture with 40% concentrate is pumped into the vat at a rate of 100 gallons per minute. With the contents of the vat constantly stirred, orange juice also flows from the vat at a rate of 100 gallons per minute. With $x(t)$ denoting the amount of concentrate and $c(t)$ denoting the percentage of concentrate in the vat at time t, we come up with the following initial value problem (IVP) as a model. The differential equation is a first-order linear equation.

```
> restart;
> OJ:= diff(x(t),t)+0.002*x(t) = 40;
```

$$OJ := \frac{d}{dt} x(t) + 0.002\, x(t) = 40$$

```
> initOJ := x(0) = 5000;
```

$$initOJ := x(0) = 5000$$

We let Maple find the exact solution.

```
> solnOJ := dsolve({OJ,initOJ},x(t));
```

$$solnOJ := x(t) = 20000 - 15000\, e^{-\frac{1}{500} t}$$

Recalling that time is in minutes and that the vat originally contains 10% concentrate, let's see what the situation is after 15 minutes.

```
> soln15:=eval(solnOJ,t=15);
```

$$soln15 := x(15) = 20000 - 15000\, e^{-\frac{3}{100}}$$

Let's get a decimal here for the number of gallons of concentrate.

```
> soln15:=evalf(rhs(%));
```

$$soln15 := 5443.31700$$

We divide by 50,000 to get the percentage of concentrate.

```
> conc15:=soln15/50000;
```

$$conc15 := 0.1088663400$$

So it looks like we're up to about 10.9%. Now find the time in minutes for the percentage of concentrate in the vat to reach 12%.

```
> conc:=rhs(solnOJ)/50000;
```

$$conc := \frac{2}{5} - \frac{3}{10} e^{-\frac{1}{500} t}$$

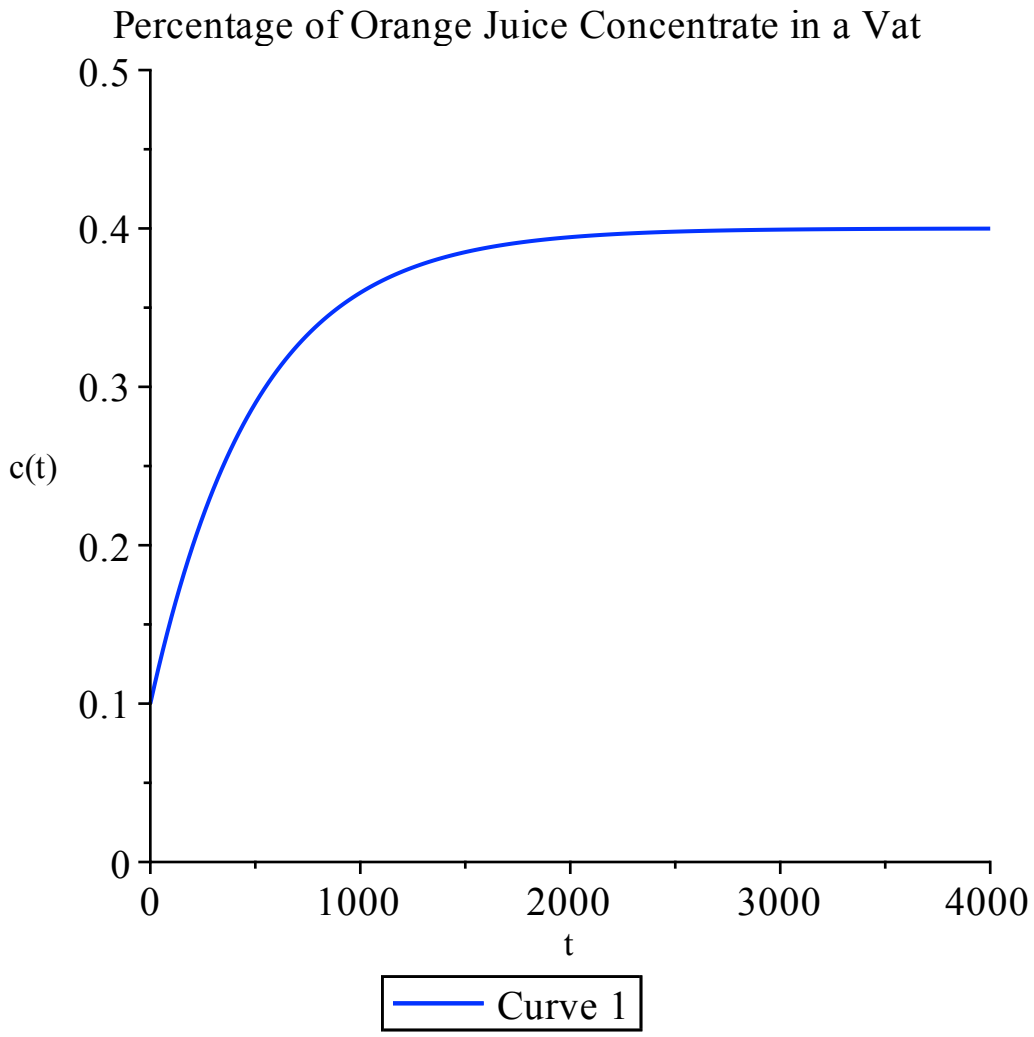
```
> solve(conc=.12,t);
```

$$34.49643574$$

In other words, we will reach a percentage of 12% concentrate in the vat in about 34.5 minutes. Let's plot the solution to see what happens as time goes on.

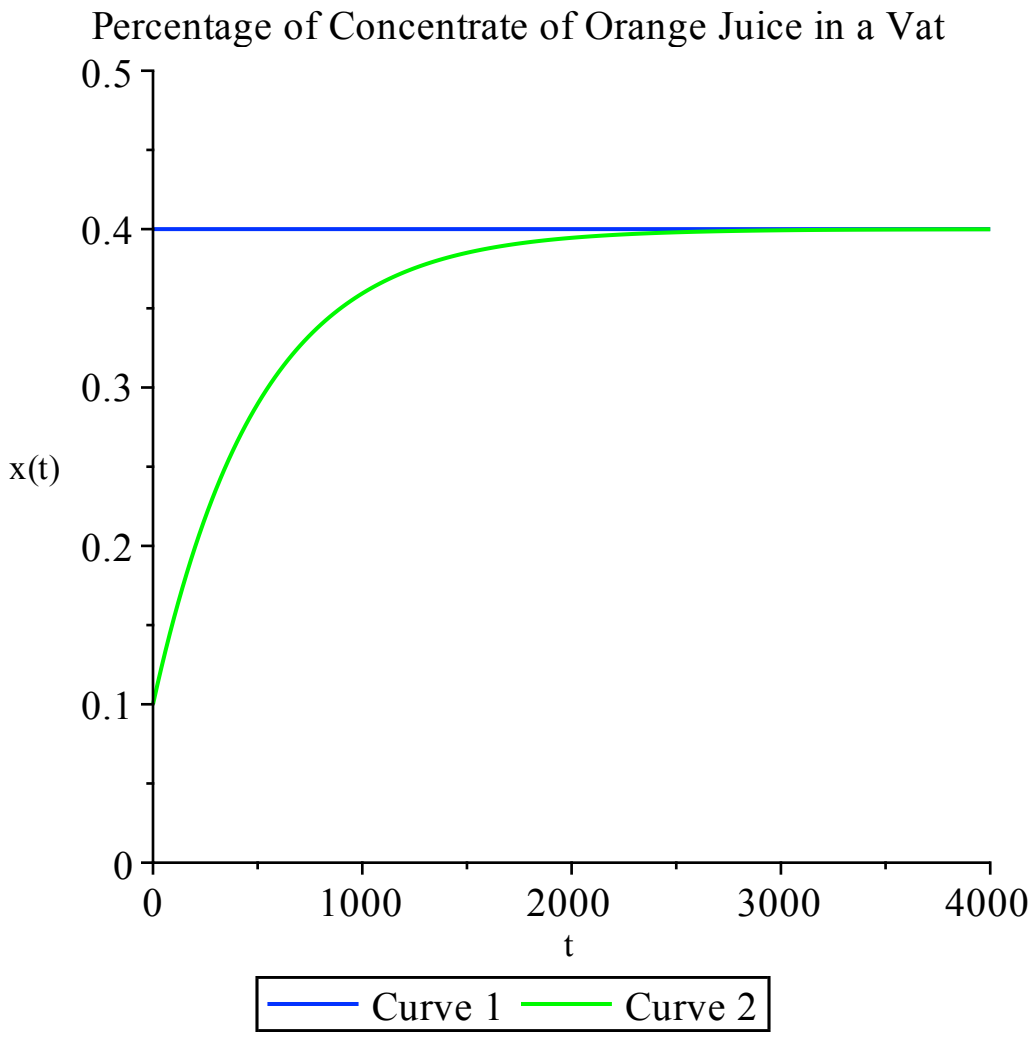
```
> plot(conc, t=0..4000,0..0.5, color=blue, labels=["t","c(t)"],title=
  "Percentage of Orange Juice Concentrate in a Vat",titlefont=[TIMES,
```

```
ROMAN,12]);
```



Observe that the percentage of concentrate in the vat is approaching a value of 40% as time evolves, the same as the percentage of concentrate flowing into the vat. This is hardly surprising, and can be clearly seen when we draw the horizontal asymptote: x = 0.4:

```
> plot({conc, .4}, t=0..4000,0..0.5, color=[blue, green],labels=["t",
  "x(t)"],title="Percentage of Concentrate of Orange Juice in a Vat",
  titlefont=[TIMES,ROMAN,12]);
```


Percentage of Concentrate of Orange Juice in a Vat
0.5
0.4
0.3
0.2
0.1
0
x(t)
0
1000
2000
3000
4000
t
Curve 1
Curve 2