Page 17, Problem 3c

We want to construct and graph the bezier curve going from point (0,0) with guidepoint (0.5,0.5) to point (4,6) with entering guidepoint (3.5,7) and exiting guidepoint (4.5,5) to point (6,1) with guidepoint (7,2).

We first look at the directions for bezier.

```
> with(numanal);
[SOR, SOR_dir, adaptpq, adaptpq_dir, bezier, bezier_dir, bisection, bisection_dir, chop, chop_dir, clamped_spline, clamped_spline_dir, divided_diff, divided_diff_dir, extrap, extrap_dir, falseposition, falseposition_dir, fixedpoint, fixedpoint_dir, gaussseidel, gaussseidel_dir, hermite, hermite_dd, hermite_dd_dir, hermite_dir, horner, horner_dir, jacobijacobidir, muller, muller_dir, natural_spline, natural_spline_dir, newton, newton_dir, romberg, romberg_dir, secant, secant_dir, steffensen, steffensen_dir]
```

Now we use that to construct the bezier curve.

```
> b:=bezier([[0,0],[4,6],[6,1]],[[.5,.5],[4.5,5]],[[3.5,7],[7,2]],b);
b := [[[0, 0], [4, 6], x_0(t) = -5.0 t^3 + 7.5 t^2 + 1.5 t, y_0(t) = -13.5 t^3 + 18.0 t^2 + 1.5 t], [[4, 6], [6, 1], x_1(t) = -5.5 t^3 + 6.0 t^2 + 1.5 t + 4, y_1(t) = 4 t^3 - 6 t^2 - 3 t + 6]]
```

We do a parametric plot of the bezier curve one piece at a time.

```
> p1:=plot([rhs(op(3,b[1])),rhs(op(4,b[1])),t=0..1],-10..10,-10..10):
p2:=plot([rhs(op(3,b[2])),rhs(op(4,b[2])),t=0..1],-10..10,-10..10):
> with(plots):
p3:=pointplot([[0,0],[4,6],[6,1]]):
display(p1,p2,p3);
```
Let's add in the guideposts.

```maple
> p4:=pointplot({[0,0],[.5,.5]},style=line,color=blue):
> p5:=pointplot({[4,6],[4.5,5]},style=line,color=blue):
> p6:=pointplot({[4,6],[3.5,7]},style=line,color=green):
> p7:=pointplot({[6,1],[7,2]},style=line,color=green):
> display(p1,p2,p3,p4,p5,p6,p7);
```
Generate Bézier Curves

A Bézier curve is a polynomial determined by a set of points in such a way that it interpolates the first and last points, but has its shape determined by the remaining points. This task allows you to interactively define the points and view the curve.

<table>
<thead>
<tr>
<th>Bézier Curves</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Use the slider to select $n$, the degree of the Bézier curve.</td>
</tr>
<tr>
<td>$n =$</td>
</tr>
</tbody>
</table>
• Press Initialize to initialize/clear the plot window.

• Click on the plot area and select the Click and Drag Manipulator ( ) from the Plot menu or plotting toolbar.

• Click to insert $n + 1$ control points $P_k, k = 0, \ldots, n$.

• Drag control points to modify the Bézier curve.

• Below, find the Bézier curve

$$
\mathbf{R} = \sum_{k=0}^{n} \binom{n}{k} (1 - u)^{n-k} u^k P_k
$$

$$
\begin{bmatrix}
2.265625 + 9.062500 u - 3.671875 u^2 - 0.5729167 u^3 \\
5.027778 + 9.000000 u - 27.16667 u^2 + 15.11111 u^3
\end{bmatrix}
$$