Maple's Numeric Integration

> restart;

We are going to look at Maple's built-in numeric integration. We begin by going to the help page for numerical integration.

> ?evalf,int;

After scanning the page, we set infolevel to 5 for both evalf/int and int and look at an example on the help page in some detail.

> infolevel[`evalf/int`]:=5;infolevel[int]:=5;

We look at a first integral.

> Int( exp(-x^3)/(x^2+1), x = 0..1 )=int( exp(-x^3)/(x^2+1), x = 0..1 );

```plaintext
int: Beginning integration with _EnvContinuous=_EnvContinuous,
    _EnvAllSolutions=_EnvAllSolutions, and _EnvCauchyPrincipalValue=
    _EnvCauchyPrincipalValue.
Definite Integration:   Integrating expression on x=0..1
Definite Integration:   Using the integrators [distribution, piecewise,
    series, o, polynomial, ln, lookup, cook, ratpoly, elliptic,
    elliptictrig, meijergspecial, improper, asymptotic, ftoc, ftocms,
    meijerg, contour]
Definite Integration:   Trying method distribution.
Definite Integration:   Trying method piecewise.
Definite Integration:   Trying method series.
Definite Integration:   Trying method o.
Definite Integration:   Trying method polynomial.
Definite Integration:   Trying method ln.
Definite Integration:   Trying method lookup.
LookUp Integrator:   unable to find the specified integral in the table
Definite Integration:   Trying method cook.
Cook LookUp Integrator:   Given Integral
Int(exp(-x^3)/(x^2+1),x = 0 .. 1)
Fits into this pattern:
Int(exp(-Ucplex*x^S1)*x^N*ln(B*x^DL)^M*cos(C1*x^R)/((A0+A1*x^D)^P),x =
t1 .. t2)

Cook LookUp Integrator:   --> but does not fit into any sub-classes
Cook LookUp Integrator:   returning answer from cook pattern 1c
Definite Integration:   Trying method ratpoly.
Definite Integration:   Trying method elliptic.
int/elliptic: trying elliptic integration
int/ellalg/elltype: Checking for an elliptic integral exp(-x^3)/(x^2+1)
freeof(x)x
Definite Integration:   Trying method elliptictrig.
Definite Integration:   Trying method meijergspecial.
Definite Integration:   Trying method improper.
Definite Integration:   Trying method asymptotic.
Definite Integration:   Trying method ftoc.
int/indef1: first-stage indefinite integration
int/indef2: second-stage indefinite integration
int/exp: case of integrand containing exp
int/indef1: first-stage indefinite integration
```
int/indef2: second-stage indefinite integration
int/exp: case of integrand containing exp
int/prpexp: case ratpoly*exp(arg)
int/rpexp: nonlinear denominator case not yet coded
int/prpexp: case ratpoly*exp(arg)
int/rischnorm: enter Risch-Norman integrator
int/rischnorm: exit Risch-Norman integrator
int/risch: enter Risch integration
int/risch: the field extensions are
\[
[x^-, e^{-x^3}]
\]
int/risch: Introduce the namings:
\[
\{ _\text{th}_1 = e^{-x^3} \}
\]
int/risch/int: integrand is
\[
\frac{-\text{th}_1}{x^2 + 1}
\]
int/risch/exppoly: integrating
\[
\frac{-\text{th}_1}{x^2 + 1}
\]
int/risch/diffeq: solving Risch d.e. \( y' + f \ y = g \) where \( f, g \) are:
\[
-3 \ x^2, \ \frac{1}{x^2 + 1}
\]
int/risch/DEratpoly: solving Risch d.e. \( y' + f \ y = g \) where \( f, g \) are:
\[
-3 \ x^2, \ \frac{1}{x^2 + 1}
\]
int/risch/exppoly: Risch d.e. has no solution
int/risch: exit Risch integration
Definite Integration: Trying method ftocms.
int/risch: enter Risch integration
int/risch: the field extensions are
\[
[x^-, e^{-x^3}]
\]
int/risch: Introduce the namings:
\[
\{ _\text{th}_1 = e^{-x^3} \}
\]
int/risch/int: integrand is
\[
\frac{-\text{th}_1}{x^2 + 1}
\]
int/risch/exppoly: integrating
\[
\frac{-\text{th}_1}{x^2 + 1}
\]
int/risch/diffeq: solving Risch d.e. \( y' + f \ y = g \) where \( f, g \) are:
\[
-3 \ x^2, \ \frac{1}{x^2 + 1}
\]
int/risch/DEratpoly: solving Risch d.e. \( y' + f \ y = g \) where \( f, g \) are:
\[
-3 \ x^2, \ \frac{1}{x^2 + 1}
\]
We see that Maple tries many things but cannot do an exact evaluation, so we now try to integrate numerically using Maple's default method.

```maple
> Int( exp(-x^3)/(x^2+1), x = 0..1 )=evalf(Int( exp(-x^3)/(x^2+1), x = 0..1 ));
```

We have success. Maple used **d01agc**, a NAG (Numerical Algorithms Group) routine that operates at hardware floating-point speed. Next we use an adaptive double exponential method. We also ask for 20 digits of precision.

```maple
> Int( exp(-x^3)/(x^2+1), x = 0..1 )=evalf(Int( exp(-x^3)/(x^2+1), x = 0..1,digits=20, method=_Dexp));
```
Now we will try an adaptive Gaussian quadrature method and, after raising Digits to 25, set $10^{-20}$ as a relative error tolerance.

> Digits := 25;

> \int_0^1 \frac{e^{-x^3}}{x^2 + 1} \, dx = 0.6649369430739274561309408