

RC CIRCUIT

OBJECTIVE: To study the charging and discharging process for a capacitor in a simple circuit containing an ohmic resistance, R , and a capacitance, C .

THEORY: Consider the circuit shown below in Fig. 1:

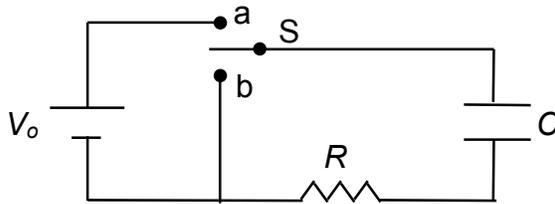


FIGURE 1

1. Charging the capacitor (capacitor is initially uncharged):

If the switch S is moved to position "a" at time $t=0$, a time dependent current, $I(t)$, begins to flow, and the charge q on the capacitor increases. We can apply Kirchoff's second rule (conservation of energy) to get:

$$V_R + V_C = V_0$$

or, using Ohm's Law ($V_R = IR$) and the definition of capacitance ($C = q / V_C$),

$$IR + q / C = V_0 .$$

Further, we can relate q to I ($I = dq / dt$):

$$R(dq / dt) + (1/C)q = V_0 .$$

This is a differential equation which we can rearrange as

$$dq / (q - V_0 C) = (-1/RC)dt ,$$

Integrating both sides, where the lower limits are $q = 0$ at $t = 0$ and the upper limits are $q = q(t)$ at $t = t$ gives

$$q(t) = V_0 C (1 - e^{-t/RC}) .$$

Since $C = q / V_C$, then $V_C(t) = q(t)/C$ and the voltage across the capacitor during charging is

$$V_C(t) = V_0 (1 - e^{-t/RC}) \quad (1)$$

Check: At $t = 0$, $V_C = 0$ since the capacitor is initially uncharged. At large t , V_C approaches V_0 .

2. Discharging the capacitor (capacitor initially fully charged):

If the switch is moved to position "b" at $t = 0$, the capacitor will discharge and a current will flow through the resistor. Again applying Kirchhoff's second law (conservation of energy) we get:

$$V_R + V_C = 0 \quad \text{or} \quad R(dq/dt) + (1/C)q = 0 .$$

This is a differential equation which we can rearrange as

$$dq/q = -(1/RC)dt$$

Integrating both sides, where the lower limits are $q = Q_0$ at $t = 0$ and the upper limits are $q = q(t)$ at $t = t$ gives

$$\begin{aligned} q(t) &= Q_0 e^{-t/RC} \quad \text{and} \\ V_C(t) &= V_0 e^{-t/RC} . \end{aligned} \tag{2}$$

Check: At $t = 0$, $V_C = V_0$. At large t , V_C approaches zero.

NOTE: The product RC is a time called the "RC time constant" or the "1/e time" and is denoted by τ . When $t = \tau = RC$, the voltage across the capacitor is 1/e times the initial voltage:

$$V_C(\tau) = V_0 e^{-1} = (0.368)V_0 . \tag{3}$$

3. Using semi-log graph paper to plot an exponential function:

Semi-log graph paper has a regular scale on the horizontal axis, but the vertical axis is set up to automatically record the base 10 log of the number. To see how this works, consider plotting the following data pairs: (1,10), (2,100), (3,1000). The x-values (1,2,3) are equally spaced. The y-values (10,100,1000) are obviously not equally spaced, but $\log(10)=1$, the $\log(100)=2$, and the $\log(1000)=3$ so the logs of the y-values are equally spaced. Note that this is true on the semi-log graph paper if you assume that the first 1 on the vertical axis is 10, the second 1 is 100, and the third 1 is 1000. Thus, if we plot our voltage (V_C) values on the semi-log scale, the graph paper will automatically plot the $\log(V_C)$.

Now, if we consider Eq. (2) and take the log of both sides, we get:

$$\begin{aligned} \log[V_C(t)] &= \log[V_0 e^{-t/RC}] \\ &= \log[V_0] + \log[e^{-t/RC}] \\ &= \log[V_0] - (t/RC) \log[e] \\ \log[V_C(t)] &= \log[V_0] - (.434/RC)t . \end{aligned} \tag{4}$$

Thus, if we plot the $\log[V_C(t)]$ vs t , we *should* get a straight line with a y-intercept of $\log[V_0]$ and a slope of $-.434/RC$. If, indeed, $V_C(t)$ vs t plots as a straight line on the semi-log graph, then we can conclude the $V_C(t)$ is an exponential function!

Part 1: Capacitance of a Capacitor

PROCEDURE:

1. Build the circuit as shown in Fig. 2 below. V_o is supplied by the Lambda Power Supply (PS); C is supplied by the capacitor marked C_1 on the board; and the DMM is to be used as the resistor, R . (It will also act as a voltmeter to measure V_C .) NOTE: *Electrolytic capacitors are used in this experiment and the red terminal of the capacitor MUST be connected to the positive voltage or the capacitor will EXPLODE!*

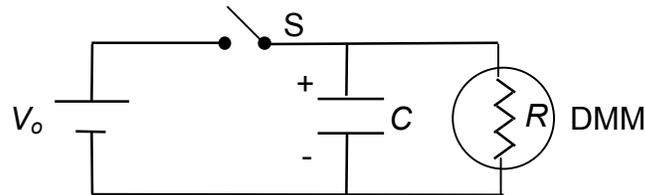


FIGURE 2

2. Close the switch, S . Set the PS voltage at 50 Volts as measured by the DMM. The capacitor will be fully charged within fractions of a second (since there is almost no resistance between it and the PS).

3. We are going to time how long the capacitor takes to discharge to $1/e$ of its initial voltage, that is, we are going to find the RC time constant, τ . To do this, first calculate the target voltage $V_o e^{-1} = (50 \text{ Volts}) \cdot (.368)$. Then as you open the switch, start the stop watch. Record the time when the voltage reaches this target voltage. This time is the RC time constant for this combination of resistor and capacitor.

4. To see if the voltage does decay exponentially with time, we will take a set of readings of the voltage across the capacitor as time increases. To do this, first close the switch to charge up the capacitor. Now open the switch and at the same time start your stop watch. Read the voltage across the capacitor, V_C , for various times, t , while the capacitor discharges through the DMM which acts like a resistor, R . The input impedance of your DMM is given as 10 MegaOhms by the manufacturer, i.e. $R = 10 \text{ M}\Omega$. Try to get as many readings as you can while the capacitor discharges. Take readings for at least two minutes (120 seconds). Tabulate your readings in a table of V_C (Volts) vs. t (sec). *Caution: When the stopwatch reads 1:00, this is NOT 100 seconds, but 1 minute and 0 seconds, or 60 seconds.*

5. Replace the C_1 capacitor used above with the capacitor labeled C_2 . Do not change any other part of the circuit. As in procedure 3, measure the time it takes for V_C to reach $(0.368)V_o$. (You do NOT have to create the table.)

REPORT:

- Using your $1/e$ time in procedure one and the given R value of $10\text{ M}\Omega$, calculate a value for the capacitance of C_1 .
- From the values of voltage and time in your data table, plot the Voltage vs Time graph for capacitor C_1 . Have the computer draw the best exponential curve to the data and display the equation of the best curve.
 - From this equation, obtain the time constant, $\tau = RC$. Is this the same (within experimental uncertainty) as the time constant you directly measured in Step 1?
 - Determine a second "measured" value of the capacitance from this time constant and compare this second value to the first value determined in Step 1.
- From the values of voltage and time in your data table, make a semi-log plot of Voltage vs. Time. (The easiest way to do this in Excel is to format the voltage axis in Plot 1. In the "Format Axis" window, choose the "Scale" tab and click on logarithmic scale. The plot should automatically be generated.) Does the plot appear linear as predicted by theory?
- Is the $1/e$ time for the second capacitor, C_2 , greater or less than the time for C_1 ? Does this indicate that C_2 is greater or less than C_1 ? Using this $1/e$ time, calculate the "measured" capacitance of C_2 .

Part 2: Parallel and Series Combinations of Capacitors

PROCEDURE:

- In this part of the experiment, use the same two capacitors as in Part 1. Build the circuit shown below in Fig. 3.

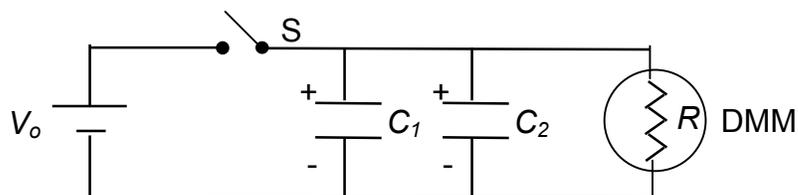


FIGURE 3

The PS supplies the V_0 and should be set at 50 Volts. The DMM will supply the resistance, R , as well as serve to measure the voltage across the effective capacitor.

- Close the switch, S , to charge the capacitors (which again should happen quickly - why?) and then open S and measure the time for the voltage to drop to $(0.368)V_0$ as the capacitors discharge through the resistance offered by the DMM ($10\text{ M}\Omega$). This time should be the time constant for this parallel combination of C_1 and C_2 and R .

3. Build the circuit shown in Fig. 4 below. Find the time constant for this series combination of C_1 and C_2 and R .

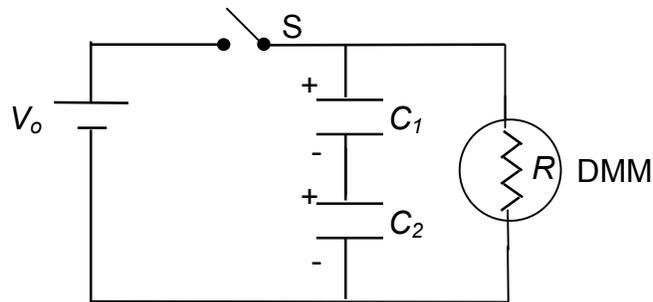


FIGURE 4

REPORT:

1. Calculate the experimental equivalent capacitance for both the parallel and the series combination of capacitors using your experimental $1/e$ time measurements. Also calculate from theory (using your "measured" values for C_1 and C_2 from Part 1) what you would expect for each. Compare your experimental results to your theoretical calculations.
2. The manufacturer claims that the nominal values of the capacitors are $C_1 = 4.7 \mu\text{F}$ and $C_2 = 2.2 \mu\text{F}$. These values are guaranteed by the manufacturer with a certain confidence level (typically to within 5-10%). Compare your "measured" values of the capacitances of C_1 and C_2 from Part 1 to these nominal values. What percent difference is there between the measured and nominal values? Is it less than 10%?

Part 3: Body Resistance

PROCEDURE:

1. Build the circuit shown in Fig. 5 below where the capacitance, C , is a large capacitor with a value that can be read off the capacitor (it should be between 80 and 250 μF). As before, the DMM will measure the voltage across the capacitor, but you will act as a resistor when you touch the leads marked a and b .

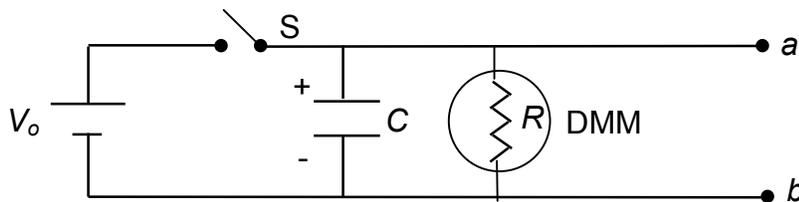


FIGURE 5

2. Do NOT touch the leads a and b until instructed to do so.

Set the PS voltage at 5.1 Volts. Close the switch, S , to charge the capacitor. Then open the switch and watch the voltage readout slowly decrease (we are initially using the DMM as a resistor). When the voltage reaches 5.0 Volts (V_o) grab the open ends "a" and "b" with your left and right hands respectively, so that the capacitor will now mainly discharge through your body instead of through the DMM. Do this with dry fingers. [CAUTION: If you grip the open ends while the switch is closed, you will have the power supply trying to run current through you instead of only the DMM.] Watch the readings on the voltmeter, and measure the time it takes the voltage to drop to $(0.368)V_o$. From this RC time constant, obtain the equivalent resistance of you and the meter in parallel using the nominal capacitance. From this equivalent resistance, calculate your body resistance.

3. Repeat Step 2 with wet fingers.

REPORT:

1. Calculate your body resistance (both with dry and wet fingers) using the times from Steps 2 and 3 and the stated value of the capacitance.
2. Compare your resistances with dry and wet fingers. Comment on any difference.

FOR ALL PARTS:

As always, include a section on experimental uncertainties: what did you actually measure, and how well were you able to measure it? How much effect does this uncertainty in measurement have on your results?