

AC CIRCUITS

OBJECTIVE: To study the effect of alternating currents on various electrical quantities in circuits containing resistors, capacitors and inductors.

Part 1: Inductance of a Coil

THEORY: If the current in a resistor R , a capacitor C , and/or an inductor L is given by:

$$I(t) = I_o \sin(\omega t) \quad \text{where } \omega = 2\pi f$$

and f is the frequency, then the corresponding voltages across each element can be written as:

$$\begin{aligned} V_R(t) &= V_{Ro} \sin(\omega t) &= R I_o \sin(\omega t) & \text{since } V_R = IR \\ V_C(t) &= V_{Co} \sin(\omega t - \pi/2) &= (1/\omega C) I_o \sin(\omega t - \pi/2) & \text{since } V_C = Q/C \text{ \& } Q = \int I dt \\ V_L(t) &= V_{Lo} \sin(\omega t + \pi/2) &= \omega L I_o \sin(\omega t + \pi/2) & \text{since } V_L = L(dI/dt) \end{aligned}$$

This means that the voltage across a resistor, V_R , is in phase with the current, I ; the voltage across a capacitor, V_C lags the current by 90° ($\pi/2$ radians); and the voltage across an inductor, V_L , leads the current by 90° . These angles (between voltage and current) are called phase angles. **NOTE:** a DMM can only give rms values ($V_{rms} = V_o / \sqrt{2}$, $I_{rms} = I_o / \sqrt{2}$); a dual trace oscilloscope can be made to show the relative phase differences.

In terms of rms values only (those measured by a DMM),

$$\begin{aligned} V_{Rms} &= I_{rms} R \\ V_{Cms} &= (1/\omega C) I_{rms} = X_C I_{rms} & \text{where } X_C &= 1/\omega C \\ V_{Lms} &= \omega L I_{rms} = X_L I_{rms} & \text{where } X_L &= \omega L \end{aligned}$$

The X_C is called the capacitive reactance; the X_L is called the inductive reactance. Even though from conservation of energy (Kirchoff's Law), $\Sigma V(t) = 0$, when we have phase differences we may have $\Sigma V_{rms} \neq 0$! To take into account the phase difference between these voltages, we can use a construct called a **phasor diagram**. We have constructed phasor diagrams for two types of circuits in the examples below (see Figs. 1b and 2b). Note that V_L and V_C are drawn $+90^\circ$ and -90° out of phase with I while V_R is in phase with I . Note also that R does not depend on frequency, X_C decreases with increasing frequency, and X_L increases with increasing frequency.

1. Series LR Circuit

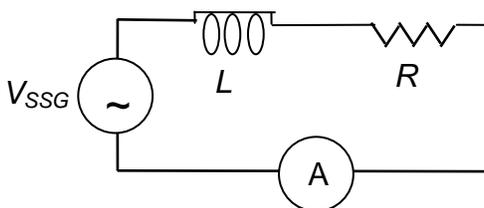


FIGURE 1a

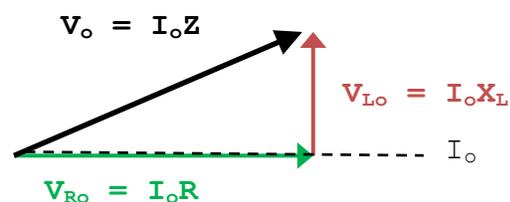


FIGURE 1b

In the above figures,

$$V_{SSG} = V_o \sin(\omega t) = (\sqrt{2}V_{SSG_{rms}}) \sin(\omega t);$$

the inductive reactance is

$$X_L = V_{L_o} / I_o = \omega L = 2\pi f L; \quad (1)$$

and the **impedance** of the circuit is

$$Z \equiv V_o / I_o = V_{SSG_{rms}} / I_{rms} = \sqrt{R^2 + X_L^2} \quad (2)$$

Note that the relation of V to I no longer depends just on R . This is due to the presence of the inductor. We need to replace R with Z , the impedance (or generalized resistance). Also note that due to the presence of the phase angle, we cannot just add R and X_L together like we could with individual resistances in series. From the phasor diagram it is clear we must use the Pythagorean theorem as in Eq. (2) above.

These relations allow us to determine the inductance in the circuit. From Eq. (2) :

$$X_L = \sqrt{(V_{SSG_{rms}} / I_{rms})^2 - R^2} \quad (3)$$

and then from Eq. (1):

$$L = X_L / 2\pi f \quad (4)$$

Thus one can set up a circuit such as Fig. 1a; measure $V_{SSG_{rms}}$, I_{rms} , R and f , and then obtain the inductance of the inductor via Eqs. (3) and (4).

PROCEDURE:

1. Measure the resistance R of the coil on your lab table with the DMM and record it.
2. Set up the circuit in Fig. 1a. R and L are the resistance and inductance of the coil. So the circuit will contain merely a power supply (sine-square wave generator SSG) and the coil. Use one DMM as an ammeter (AC mode at the 200 mA range) connected in series, and use a second DMM as a voltmeter (AC mode at the 20 V range) connected in parallel across the SSG outputs.
3. Set the SSG voltage (using the AMPL knob) near maximum and the SSG frequency to near 500 Hz. (You will need to select **sine wave** button and the 1 kHz range button and then adjust the FREQUENCY knob.) Check the frequency using the O-scope. [Recall the oscilloscope can directly measure the period, T , of a sine wave, not its frequency, f . But also recall that $f = 1/T$.] Record this frequency.
4. Now measure and record the SSG voltage and the current in the circuit using your two DMM's. Remember that the DMM readings in the AC mode will be **rms** values.
5. From the values measured and recorded in the previous steps ($V_{SSG_{rms}}$, I_{rms} , R , f) obtain the **experimental inductance**, L of the coil using Eqs. (3) and (4).
6. Change the frequency to near **750 Hz**, and repeat steps 4 & 5.
7. Then change the frequency to near **1,000 Hz**, and repeat steps 4 & 5.

8. Now average the three values you have obtained for L to get the best **experimental** value for the inductance of the coil.

9. The inductance of a long solenoid (length \gg radius) is given by $L_{long} = \mu_o N^2 A / \ell$

where $\mu_o = 4\pi \times 10^{-7}$ Wb/Am, N = number of turns, ℓ = length, and A = cross sectional area = πr^2 where r = radius. However, in our case $r \approx \ell$. It can be shown that for a short solenoid, L is smaller by a factor K where K is a function of the ratio r/ℓ .

$$L_{short} = KL_{long} = K(\mu_o N^2 A / \ell)$$

Representative values of K are given in the following table:

r/ℓ	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0	4.0	10.0
K	1.00	0.85	0.74	0.65	0.58	0.53	0.43	0.37	0.24	0.1

(Values in the table are from "Electromagnetic Fields and Waves" by Lorrain & Corson)

Since the coil is very thick, there is not one obvious value for r . To take this into account,

- first calculate a geometric value for L based on the inside radius, and then
- calculate a second value for L based on the outside radius.

This will then give us a **range** for the **geometric** value of L .

REPORT:

- Record your data and include all units.
- Show sample calculations for each type of calculation made.
- Report your individual experimental values for L (steps 5, 6, and 7) and your average experimental value for L (step 8), and see how close each of the individual values is to the average. Discuss any major reasons for the discrepancies.
- Report your geometric range of values for L using the procedure in step 9, and compare to the experimental value obtained in step 8. Are your experimental values for L within the range of your geometric values for L ? Should they be? Which value do you consider the best value of L for your coil? You will need this best value for L in parts 2 and 3 below.

Part 2: Series Resonance

Series LRC Circuit

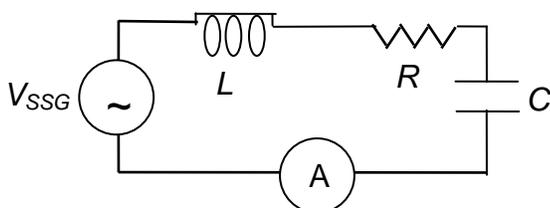


FIGURE 2a

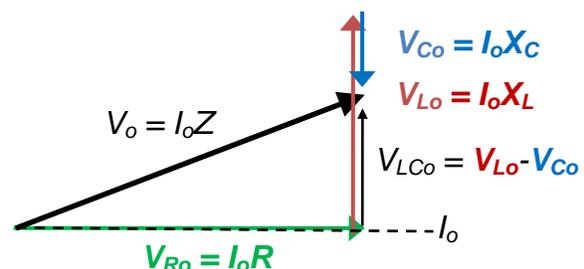


FIGURE 2b

THEORY:

For the series connection of a resistor, inductor, and capacitor, we get (using the phasor diagram):

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad (5)$$

and

$$I_{rms} = V_{SSGrms} / Z = V_{SSGrms} / \sqrt{R^2 + (X_L - X_C)^2} \quad (6)$$

This shows that the current has its **maximum** value when $X_L = X_C$, i.e. $\omega L = 1 / \omega C$ or when

$$\omega = \omega_o = 1 / \sqrt{LC} \quad (7)$$

In this case we speak of series **resonance**, and the frequency at which this resonance occurs is called the **resonance frequency**, f_o , given by:

$$f_o = 1 / 2\pi\sqrt{LC} \quad (8)$$

PROCEDURE:

- Set up the circuit of Fig. 2a with the coil, decade capacitor, and ammeter connected in series and the voltmeter connected in parallel across the outputs of the SSG, with $C = 1 \mu\text{F}$, L = the inductance of the coil (which you determined in Part One), and R = the resistance of the coil. Do not yet turn on the SSG.
- Consider Fig. 2b and Eqs. (5-8). Calculate the **theoretical resonance frequency** of the circuit based on your values for L , C and R using Eq. (8).
- Now obtain the **experimental** resonance frequency by turning on the SSG and adjusting the frequency until the current reaches its maximum value and the voltage reaches its minimum value. (*Note: away from resonance, the voltage should not change very much, but near resonance, the SSG may be near its maximum power output and may not be able to increase the current much, but you should see the voltage fall as you approach resonance.*) When the current reaches maximum and voltage minimum, then Z reaches its minimum value and $(1/Z)$ reaches its maximum value. Check this resonance frequency by measuring the period, T , of the wave with the O-scope and then calculating the frequency, f [recall $f = 1/T$].
- Repeat steps 2 & 3 for two other values of C such as $0.1 \mu\text{F}$ and $0.01 \mu\text{F}$.
- Set $C = 1 \mu\text{F}$ and the SSG voltage at 2 V. We will now observe how **sharp** the resonance peak is by looking at I and V values for frequencies on either side of your **experimental** resonance frequency. To do this,
 - lower** the frequency of the SSG by about ten Hertz from the resonance frequency using the frequency shown on the SSG,
 - adjust the SSG voltage to remain at 2 V,
 - record f , I_{rms} and V_{SSGrms} .
 - Again lower the frequency another 10 Hz and adjust the voltage to 2 V; then record your f , I and V .
 - Do this one more time at another 10 Hz interval.
 - Do this two more times at 20 Hz intervals.
 - Now repeat the process but this time **increase** the frequency from f_o instead of decreasing it. Remember to adjust the voltage to 2 V after adjusting the frequency each time.

6. With the data obtained from step 5, go to one of the computers in the lab and call up the *RLC* program from the lab menu. Enter in your data and the program will plot the graph of $(1/Z)$ versus frequency. It will plot a theoretical curve based on your values of C , L and R , and will also place your experimental data points on the graph. See how well the theoretical curve and the experimental points agree. Also see if you can adjust the values of C , L and/or R to better fit the curve to your experimental values.

7. Calculate the quality factor Q using Eq. (9) (using Δf and f_o) as well as Eq. (10) (using f_o , L and R). Compare the two values for Q .

8. Now **add a resistance of $R = 200 \Omega$** to the circuit in series with the coil (thus making the total resistance about 263Ω), and repeat steps 5, 6 & 7.

REPORT:

1. Show your calculations for the predicted resonance frequency for each capacitance used.
2. Compare the predicted resonance frequency [Eq. (8)] with the value you obtained experimentally in step 3 for each of the three capacitances used. Discuss experimental errors that might account for any discrepancies.
3. Discuss the graph you made in step 6 with your instructor. On this graph be able to locate the resonance frequency and indicate the points you used in determining the experimental value [Eq.(9)] for the quality factor, Q . Do this for each of your two circuits ($R=63 \Omega$ and $R=263\Omega$).
4. Show your two values for Q [experimental from Eq.(9) and predicted from Eq.(10)] for each of your two circuits, compare them, and discuss the reasons for any discrepancies between them. What does the value of Q indicate? Which circuit has the smaller Q ? Should it? Why?
5. Why did the SSG voltage change as the frequency was varied? [HINT: the SSG has an internal Resistance]

Part 3: Filtering (e.g., radio transmitter/receiver)

THEORY: Quality Factor and Filtering

Consider the plot in Fig. 3 of I_o/V_o (which is equivalent to $1/Z$) for the *LRC* series circuit.

The sharpness of the maximum of this curve (which means a maximum in the current delivered to the circuit) can be described by the quality factor Q which is defined as:

$$Q = f_o / \Delta f \quad (9)$$

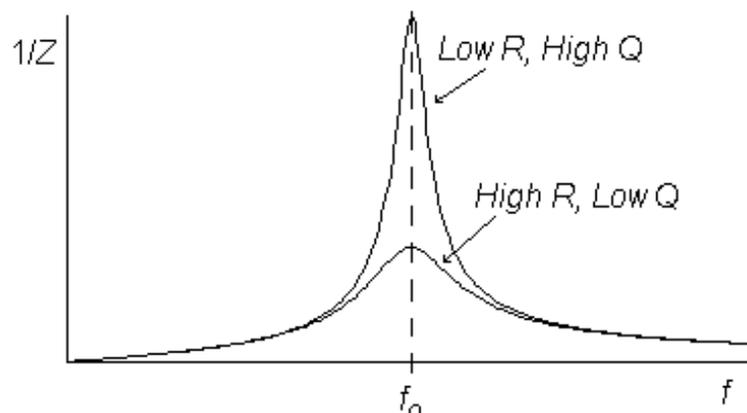


FIGURE 3

where Δf is the **bandwidth** of the circuit which is found by finding the difference between the two values of f for which the power delivered to the circuit is one-half of the maximum power (delivered at resonance). Since the power delivered is proportional to $(1/Z)^2$, the bandwidth is the difference between the two frequencies for which $1/Z$ equals $1/\sqrt{2}$ times the maximum value of $1/Z$ (at resonance). Note that a smaller resistance in the circuit means a smaller bandwidth, a larger Q , and a sharper resonance. In fact, one can show that for the series resonance studied in this lab exercise

$$Q = 2\pi f_0 L / R \quad (10)$$

Further, we find that a **series LRC circuit** (Fig. 2a) presents a low impedance to currents near the resonance frequency. Therefore, a series *LRC* circuit can act as a filter by rejecting or enhancing particular frequencies.

One of the uses of the resonant circuit is filtering. One of the most common examples is that of a radio which selects the radio station you wish to hear while filtering out all the other radio stations. In this part of the experiment we will build a "transmitter" and a "receiver". Our transmission will be via the mutual inductance of two coils. [Recall that in mutual inductance, one coil will have a current that produces a magnetic field, and that changing magnetic field will induce a voltage (and hence a current) in a neighboring coil.] Our transmitter will consist of a sine-square wave generator (to provide the energy) hooked up to a capacitor and inductor coil (to provide the resonance to get a large current). Our receiver will consist of a capacitor and inductor coil, where the coil will "pick up" the magnetic field oscillations from the transmitter coil.

NOTE: This procedure will be done in groups of 4 instead of 2.

PROCEDURE:

1. Connect the sine-square wave generator (SSG) up to a series *LRC* circuit with the coil providing both the resistance and the inductance (which you have previously determined). Use the decade capacitance box that has a range of 0 to 1.1 μF . Include an ammeter in series in the circuit so we can measure how much current is flowing.
2. Set the capacitance on the decade capacitance box to 0.9 μF . Now calculate what the resonance frequency for this series *LRC* circuit should be.
3. Hook up a dual-trace oscilloscope in parallel across the SSG [use channel 1 for this]. By changing the frequency of the SSG and noting how the current in the circuit responds, experimentally measure the frequency that gives the maximum current in the circuit. Compare this measured value of the resonant frequency to the calculated value. The coil in this circuit is to be our "transmitter". Set the frequency to the experimental resonant frequency.
4. Now place an iron or steel bar inside the coil. Observe what happens to the current reading. Experimentally determine the resonant frequency by finding the SSG frequency that gives the maximum current. Has the resonant frequency changed? Has it increased or decreased? Why? Remove the metal bar and reset the frequency to the original resonant frequency.
5. Connect another decade capacitance box (0-1.1 μF) in series with a second coil and an ammeter. This circuit will be our "receiver".

6. Set the value of the capacitance of the "receiver" to the same value as that in the "transmitter". Place the coil of the "receiver" near to the coil of the "transmitter" (in such a way that anything stuck through one coil will also be able to go through the other coil). Hook up the oscilloscope [use channel 2] in parallel with the coil of the receiver.
7. By changing the value of the capacitance of the "receiver" and noting how the current in the "receiver" changes, show that the "receiver" can be "tuned" to the frequency of the "transmitter".

[Note: Due to slight differences in the coils and in the capacitors of the decade capacitance box, the capacitances of the two circuits may be a little different.]
8. Verify from the oscilloscope that the frequency of the voltage (and hence current) of the receiver are the same as that of the transmitter.
9. Take the capacitance out of the transmitter circuit. Does the transmitter still transmit?
10. Leaving the capacitor out of the transmitter, hook up a second SSG in parallel to the first SSG. Set the frequency of the second SSG about a factor of 10 higher than the frequency of the first. Turn up the voltage on the second SSG and watch the oscilloscope to see the addition of these two voltages. Also watch the voltage of the "receiver" circuit. Does the voltage from the "receiver" circuit show the same waveform as the "transmitter" circuit? Discuss why not, particularly in terms of resonance and "filtering".
11. Can you now adjust your "receiver" to become "tuned" to the higher of the two frequencies of the "transmitter"? Try to do this, and record your procedure and results, and discuss your results in terms of filtering. [Note: As you get close to the second frequency, you may run out of adjustment room on the capacitance of the "receiver" circuit. When this happens, see if a slight change of the higher transmitter frequency will bring you in closer to true resonance.]
12. In a radio receiver, you want a high Q circuit to filter out all but the one radio station you want to listen to. You can "tune" the radio to this station by either changing C or L . For speakers, do you want a high Q filtering circuit or not? Discuss this.

REPORT:

Describe what you did in each of the steps, state your results, and discuss your results in terms of filtering.