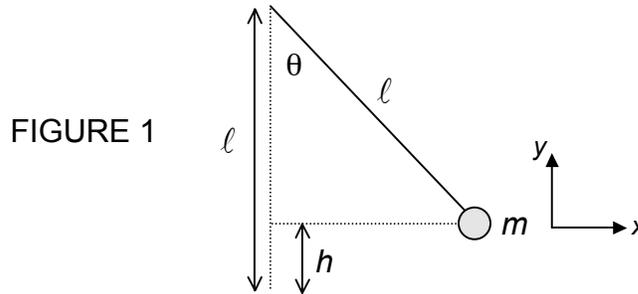


# OSCILLATIONS

**OBJECTIVE:** To study the oscillations of a pendulum and to determine what affects the period ( $T$ ) of the pendulum.

**THEORY:** The period of the pendulum is the time it takes for the bob to go from some initial  $\theta_0$  through  $\theta = 0$ , up to some negative  $\theta$ , and then back through  $\theta = 0$  until it finally reaches its original value,  $\theta_0$ . So to determine the period,  $T$ , we first need to determine how  $\theta$  behaves with time, i.e., we need  $\theta$  as a function of time,  $\theta(t)$ . This problem can theoretically be attacked either from (a) Newton's Second Law ( $\Sigma \vec{F} = m\vec{a}$ ,  $\Sigma \tau = I\alpha$ ) or from (b) Conservation of Energy.



(a) Newton's Second Law requires that we look at all the forces acting on the pendulum. Consider Fig. 1. There is gravity ( $W = mg$ ) acting down, there is the tension in the string ( $T_s$ ) acting along the direction of the string, there is air resistance acting in the direction opposite the velocity ( $R = -bv^2$  is one possible model), and there may be friction at the top of the support (which we neglect here). This is a two-dimensional problem, so:

$$\Sigma F_x = -T_s \cos(90^\circ - \theta) - b v_x^2 = m a_x \quad (1x)$$

$$\Sigma F_y = +T_s \sin(90^\circ - \theta) - mg - b v_y^2 = m a_y \quad (1y)$$

These equations are not easy to solve in this form. We can simplify these equations if we choose to ignore air resistance (just set  $b=0$  in the above equations). But we still have the problem of how to deal with  $\theta$  since  $\theta$  depends on  $x$  and  $y$  [from the geometry we see that  $\theta = \tan^{-1}(x/y)$ ] which is part of the argument of the sine and cosine functions.

A better approach might be to note that, if we assume the string does not break or stretch, then this condition can be stated as  $x^2 + y^2 = \ell^2$  and this leads to a circular motion type problem involving the single variable,  $\theta$ , instead of the two variables  $x$  and  $y$ . We can write that  $v = \ell \omega$  where  $\omega$  is the angular velocity of the bob. If we again neglect any frictional effects at the support but keep the air resistance term, we can then analyze the rotation of the pendulum in the presence of the torques provided by gravity and air resistance. The air resistance force is perpendicular to the string's direction since it is opposite the velocity direction. The lever arm of its torque is thus  $\ell$ . Gravity provides a torque with a lever arm of  $\ell \sin \theta$ . Thus, Newton's Second Law gives

$$\begin{aligned} \Sigma \tau &= -mg \ell \sin \theta - b v^2 \ell = I \alpha . \\ -mg \ell \sin \theta - b \ell^3 \omega^2 &= I \alpha \end{aligned} \quad (2)$$

(Again, we can set  $b=0$  if we wish to ignore air resistance.)

Since  $\omega = d\theta/dt$  and  $\alpha = d\omega/dt = d^2\theta/dt^2$ , we obtain a second order differential equation for  $\theta(t)$ :

$$-m g \ell \sin\theta - b \ell^3 (d\theta/dt)^2 = I (d^2\theta/dt^2) \quad (3)$$

Note that  $\theta$  is inside the argument for the sine function. The solution of this differential equation is beyond the math background assumed for this course.

(b) Conservation of Energy requires that we consider all the forms of energy in the system: potential energy due to gravity:  $mgh$ , kinetic energy:  $\frac{1}{2} m v^2$ , and any energy lost to friction. Referring to Fig. 1,  $h = \ell - \ell \cos\theta$ ,  $v = \omega \ell$  (circular motion with  $\ell$  acting as the radius), and  $E_{lost}$  depends on the angular speed,  $\omega$ , in a non-trivial way which depends on the integral of the frictional force over the distance traveled (or the integral of the frictional torque over the angle swept out):

$$mg\ell [1 - \cos\theta_i] + \frac{1}{2} m \omega_i^2 \ell^2 = mg\ell [1 - \cos\theta_f] + \frac{1}{2} m \omega_f^2 \ell^2 + E_{lost}(\omega) \quad (4)$$

If we ignore air resistance, we can drop the  $E_{lost}$  term. But even this simplifying assumption does not make this equation simple since  $\theta$  is in the argument of the cosine function; and since  $\omega = d\theta/dt$ , this gives us a first order differential equation for  $\theta(t)$  whose solution is also beyond the math background assumed for this course. But if we do ignore friction, it does allow us to draw an energy plot of potential energy and total energy vs.  $\theta$  to try to find a qualitative description of the motion.

In both cases, the theoretical analysis does not give us an easy way of calculating the period.

## Part 1: Finding an experimental relationship

### PROCEDURE:

You are provided with a support, string, weights to be used as the pendulum bob, a protractor and a stop watch.

- a) Make different pendulums and determine the period of each. Questions to consider:
  - (1) Where do you measure the length of the pendulum from: from the top of the bob, the center of the bob, the bottom of the bob, or the center of gravity of the bob?
  - (2) Should you make one timing of one oscillation, make several timings of one oscillation, make one timing of several oscillations, or make several timings of several oscillations?
  - (3) Does resistance play a significant part in the oscillations? Does resistance cause the same or different effects for different pendulums? Can you explain this? HINT: see how the angle gets smaller after successive oscillations and record your results.
  - (4) What are the major sources of experimental uncertainty in your measurements?
  
- b) See if you can experimentally determine how the period of the pendulum depends on the various parameters that you can control [e.g., you can control the mass of the pendulum bob, but you cannot control the gravity in the room]. HINT: make graphs of  $T$  vs each parameter you test. To see the relative sizes of the effects that each parameter has on the period, make your  $T$  axis identical for each plot.
  
- c) By analyzing your data, design a pendulum with a period of (1) 0.1 sec., (2) 1.0 sec., (3) 10.0 sec. If possible, check your predictions by making a pendulum with each of these periods.

## Part 2: Numerical Analysis

a) We will use the computer in this part. If the computer is not on, turn it and the monitor on. Load the Physics Menu. From the menu, type **L** to bring up the **Lab** menu and press **4** to run the oscillations lab routine. Then follow directions. [Set  $b=0$  for now. We will see it's effect below.] Determine the numerical prediction for the period of a couple of your experimental pendulums. Once you see how well the computer predicts the results of your experiments, determine the numerical prediction for the period of each of your three designs for Part 1(c) above.

*NOTE: The numerical method works by entering in the initial conditions (here:  $m, l, \theta_0, \omega_0$  and  $b$ ), and choosing a small time interval  $\Delta t$ . The time interval should be  $1/10$  to  $1/100$  of the expected period,  $T$ . The program assumes that  $\omega_0 = 0$ . From these values, the computer uses Eq. (2) to calculate  $\alpha$ , then assumes this  $\alpha$  is constant over the  $\Delta t$  and calculates a new  $\omega$ , then a new  $\theta$ , and then updates the time. With the new values of  $\theta$  and  $\omega$ , the program iterates the above procedure. The computer recognizes a period has passed when  $\omega$  changes sign the second time.*

b) Using the numerical routine above, determine a value for  $b$ , the air resistance coefficient, by watching how the angle deteriorates over time: that is, try to match your experimental observations of how the maximum angle of the pendulum decreases after each swing with your numerical results from trying different values of  $b$  in the numerical routine that predicts the value of the period (and also the angle at the end of each period). If you are successful in getting one value of  $b$  that works for one length and initial angle, see if the same value of  $b$  will also work for a different length and initial angle. Does this suggest that the value of  $b$  depends on any or all of the parameters of the system?

c) Try to obtain a "nice" formula to predict (at least approximately) the period of any pendulum based on the relevant parameters of the system.

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### FOR A HIGHER GRADE:

The gravitational potential energy of the bob can be expressed as a function of angle  $\theta$  as  $PE = mg\ell [1 - \cos\theta]$ . Plot a graph of potential energy vs.  $\theta$ , and describe the motion for various values of  $E_{total}$ . Compare these motions to what you observed in the lab.