Newton's Second Law
(without friction)

OBJECTIVE: To consider various forces and how they affect the motion of objects, particularly the motion of a cart moving along a ramp.

THEORY: One of the fundamental laws of nature is that forces do NOT cause motion, $\sum \vec{F} \neq m\vec{v}$, but rather that forces cause changes in motion (accelerations),

$$\sum \vec{F} = m\vec{a}.$$

Notes:

1. The change in motion is due to the SUM of the forces, not to each individual force.
2. This is a vector equation which means that we must break the vectors into rectangular components. In this experiment we will be dealing with vectors and motion in only 2-D rather than the full 3-D (actually, we are dealing in 3-D but all of our forces and all of our motion will only be in 2-D). For this we can use either *x* and *y* rectangular components, or we can use // and ⊥ (parallel and perpendicular) components.
3. In this experiment we will be dealing with two objects (the cart and the balancing mass) as shown in Fig. 1, rather than with just one object. In this case we will apply Newton's Second Law to each object individually. We will then have to consider how these two objects and their motions are related.

In this experiment we will deal with three kinds of force: weight (W), contact force (also called the normal force) (F_c), and tension (T). (In the next experiment, we will consider adding friction to the mix.)

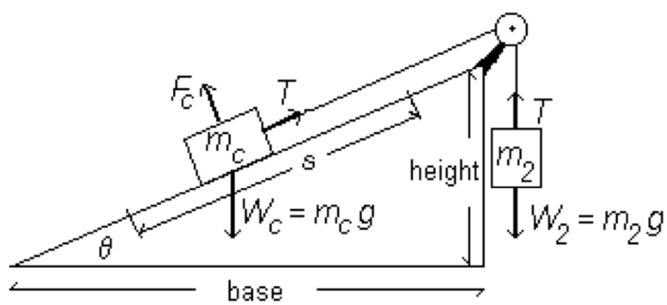

FIGURE 1

1. Consider the cart first: $\sum \vec{F}_{cart} = m_c \vec{a}_c$; Since the cart will only accelerate in the parallel direction and not the perpendicular, we will choose to use the parallel and perpendicular components, with $a_{c-//} = a$ and $a_{c-\perp} = 0$. There are three forces acting on the cart: the contact force, F_c, which acts strictly in the perpendicular direction; the tension in the string, T, which acts strictly in the parallel direction; and the weight of the cart, $W_c = m_c g$, which acts down. We therefore need to break the weight into // and ⊥ components. From geometry we get: $W_{c-//} = m_c g \sin\theta$, and $W_{c-\perp} = m_c g \cos\theta$. Putting everything together, we have:

$$\Sigma F_{c-//} = T - m_c g \sin\theta = m_c a \text{ ; and} \tag{1}$$

$$\Sigma F_{c-\perp} = F_c - m_c g \cos\theta = 0 . \tag{2}$$

2. Next, consider the balancing mass. Since it only can move up and down, we have in this case a 1-D situation. We also have only two forces: the tension in the string, T, which is the same tension as is on the cart; and the weight of the balancing mass, $W_2 = m_2 g$. As far as the acceleration goes, we note that when the cart accelerates up the ramp, the balancing mass accelerates down, so we can say: $a_2 = -a$. Hence, Newton's Second Law for this mass gives:

$$\Sigma F_2 = T - m_2 g = m_2(-a)\ . \qquad (3)$$

3. We now note that Eqs. (1) and (3) give us two equations for two unknowns: (T and a). If we eliminate T, we have:

$$a_{th} = g\,(m_2 - m_c \sin\theta)\,/\,(m_2 + m_c)\ . \qquad (4)$$

We can now solve for the "balancing mass", $m_{2\text{-}balancing}$, necessary for a_{th} to be zero so that the block does not move:

$$m_{2\text{-}balancing} = m_c \sin\theta. \qquad (5)$$

We can now use Eq. (5) to adjust Eq. (4) to read:

$$a_{th} = g\,(m_2 - m_{2\text{-}balancing})\,/\,(m_2 + m_c)\ . \qquad (6)$$

PROCEDURE:

We'll investigate situations where the cart doesn't move (static equilibrium) and where it does move. We'll test the validity of the theory developed above in both situations.

1. (a) Set the ramp at an angle of 25° by adjusting the height of one end of the board above the table by adjusting the height of the supporting crossbar. (b) Estimate the uncertainty in your angle due to the uncertainties of the values you use to try to make the angle 25°.
(c) Weight the cart on a scale and record its mass. (d) Tie a string to the cart and pass the string over the pulley and either attach a weight-holder or tie a loop for hooked weights. (e) Now add weight (mass) until the cart is balanced and neither rolls up the ramp nor rolls down the ramp (the cart being initially at rest). Call this weight W_{25} ($W_{25} = m_{25} g$). (e) Determine how much weight can be added or subtracted and still keep the cart from rolling: we'll call this uncertainty δW_{25}.

1-1. Record m_{25} and W_{25}, δm_{25} and δW_{25}, and record the ramp angle (25°).

1-2. From Newton's Second Law, calculate the balancing weight from theory (see Eq. (5) above and recall that $W = mg$).

1-3. See if the theoretical balancing weight falls within the range of W_{25} +/- δW_{25}.

2. Keep the setup from Step 1 with the following exception. Weigh the external mass and add it to the cart. Repeat the rest of Step 1 (that is, find the balancing weight).

2-1. Since the mass of the cart approximately doubles by adding the external mass, does the balancing mass (and hence weight) also approximately double?

2-2. Does theory predict that the balancing mass should double?

2-3. Calculate the theoretical balancing weight.

2-4. Does the theoretical balancing weight fall within the experimental range?

2-5. Does the uncertainty δm also double?

3. Remove the external mass. Now <u>change the angle</u> from 25° up to 50°. (This doubles the angle.) Place weights on the weight-holder until the cart is balanced. Call this W_{50}. Determine how much weight can be added and still keep the cart from rolling: we'll call this δW_{50}.

3-1. Record the mass of the weight that balances the cart on the ramp, the δm and δW, and the ramp angle θ (50°).

3-2. Since the angle is doubled, does the balancing mass also double?

3-3. Does theory say the balancing mass should double?

3-4. Calculate the theoretical balancing mass and see if the theoretical value falls within the experimental range.

4. Now <u>change the angle</u> from 50° down to 12.5°. (This angle is one-half of the initial 25°.) Use the cart without the external mass. Now place weights on the weight-holder until the cart is balanced. Call this $W_{12.5}$. Determine how much weight can be added and still keep the cart from rolling: we'll call this $\delta W_{12.5}$.

4-1. Record the mass of the weight that balances the cart on the ramp, the δm and δW, and the ramp angle θ (12.5°).

4-2. Since the angle is half of the original, is the balancing mass also half of the original?

4-3. Does theory say the balancing mass should be half?

4-4. Calculate the theoretical balancing mass and see if the theoretical value falls within the experimental range.

5. Reset the angle to 25°. <u>Add</u> between 30 and 50 grams to the balancing mass from Step 1, and then measure the time it takes the cart to go from rest up a measured distance s (s should be ~80 cm). Repeat at least two more times and find the average travel time. Call the average time t_1. Use this average in the following calculations.

5-1. Record the mass, m_2; the experimental distance, s; and the experimental time, t_1. Estimate how much uncertainty there is in the time measurement, δt_1. [Note: the experimental uncertainty in the distance is here incorporated as part of the uncertainty in the time.]

5-2. Calculate the theoretical acceleration of the cart using Eq. (6).

5-3. Using this theoretical acceleration (which is constant), the distance and the initial speed (which should be zero), calculate the theoretical time that should have resulted, t_{th1}.

5-4. See if this theoretical time falls within the range of the measured time: t_1 +/- δt_1.

6. Add the external mass to the cart and repeat Step 5, i.e. add the same amount of mass to the balancing mass as you did in Step 5 and measure the time it takes the cart to accelerate from rest through s. Do three measurements and call the average time t_2.

6-1. Answer all the questions asked in Steps 5-1 through 5-4 .

6-2. Is t_2 more, the same, or less than t_1? Can you explain this result?

<u>REPORT:</u>

1. Answer all the questions posed in the Procedure steps.
2. Are all the experimental results within the range allowed by experimental uncertainty?
3. Are there other experimental uncertainties that we did not explicitly take into account? What are they?