

CENTRIPETAL FORCE

OBJECTIVE: To investigate centripetal force and verify Newton's Second Law.

THEORY: A body moving with constant speed in a circular path is constrained to its path by a resultant force constant in magnitude and directed toward the center of the circle (See Fig. 1). This centrally directed net force acting on the body is called centripetal force. It provides the centripetal acceleration which keeps the body moving in a circle according to $\vec{F}_c = m\vec{a}_c$. In this experiment, it is supplied by a spring attached to the body (note that the weight and the contact force with the cage cancel each other out).

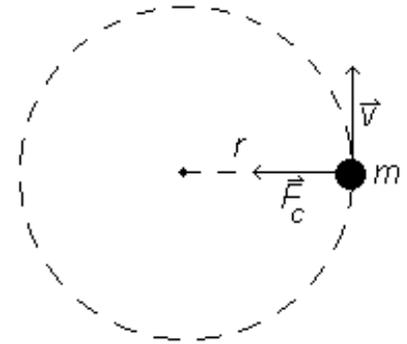


FIGURE 1

According to Newton's Third Law, the body will react to this force from the spring by exerting an equal and opposite force on the spring. Sometimes if the acceleration of the body is not realized (i.e. you are not aware of the circular motion), the ma_c term is mistakenly called the centrifugal force (a “pseudoforce”).

When the ma_c term is carried to the other side of the Newton’s Second Law

equation, $\sum \vec{F} = m\vec{a}$, its sign changes and it appears to be a force in the opposite direction from the real centripetal force. This *apparent* force directed out from the center of the circle is termed the centrifugal force. In this case, the so-called centrifugal force is not really a force, just a misplaced centripetal force term of size ma_c . (Note that the centripetal force is equal in magnitude but opposite in direction from the so-called centrifugal force.)

The magnitude of the centripetal force is related to the mass and speed of the moving body and the radius of the circular path (measured to the center of gravity of the body):

$$\sum \vec{F} = \vec{F}_c = m\vec{a}_c \quad (\text{Newton's second law}) \quad (1)$$

where

$$a_c = v^2 / r \quad (\text{circular motion}) \quad (2)$$

where v is the linear velocity tangent to the circle, and r is the radius of the circle (See Fig. 1).

If f is the frequency (number of revolutions per time) then $v = (\text{circumference of one circle}) \times (\text{number of circles per time})$, that is

$$v = 2\pi r f \quad (\text{circular motion}) \quad (3)$$

Therefore:

$$F_c = ma_c = m(v^2 / r) = m(4\pi^2 r^2 f^2 / r),$$

or

$$F_c = 4\pi^2 f^2 r m \quad (4)$$

All of these terms (F , f , r , and m) are easily measured.

Part 1. Testing Newton's Second Law: $\sum \vec{F} = m\vec{a}$

PROCEDURE 1:

- 1) Attach the centripetal apparatus securely to the rotator.
- 2) Tighten the holder with the key attached to the cord. Push the frequency button so that the display reads the frequency in rev/min (rpm). Slowly increase the speed until the pointer goes above the button. Slowly decrease the speed until the pointer goes below the button. Record the frequency for which the pointer is definitely high, for which it is definitely low, and then for which the pointer points right at the middle of the button. This will give the frequency as well as the amount of possible uncertainty in the frequency measurement, $f \pm \delta f$.
- 3) Record the mass of the moving body, m , that is stamped on the cylindrical body.
- 4) Next remove the apparatus, and hang it by a piece of string from the stand. The spring should be near the top. Hang weights on it to stretch the spring until the pointer is again opposite the button. The tension in the spring is now the same as was exerted on the body when it was rotating. The spring force now equals the weight of all the mass M attached to the end of the spring ($F = Mg$). Be sure to include in the value of M the mass of the rotating body, m . Estimate the uncertainty δM in this value of M by seeing how much mass you can add and remove to make the pointer definitely move off the center of the button.
- 5) While it is hanging, measure r from the center (white line) to the line on the body (which marks the center of gravity). Be sure to estimate the uncertainty δr in this measurement.

REPORT:

1. Now we are in a position to check Newton's Second Law. Since we now know f , r , and m , we can calculate ma_c . Do so.
2. The tension in the spring (which is equal to Mg from 4) in the above procedure) provides the centripetal force F_c . Calculate the value of F_c .
3. Does $F_c = ma_c$? Find the percent difference using the F_c value as the standard.
4. Consider your measurement uncertainties, and comment on whether they are sufficient to account for the percent error found in step 3 above. To do this, consider the percent uncertainty for each measurement: $\% \text{ uncertainty} = (\delta/\text{value}) * 100$. When quantities are multiplied, their percent uncertainties are added. Thus, when a quantity is squared (as frequency is in the ma_c calculation), its percent uncertainty is doubled.

PROCEDURE 2:

Now readjust the tension in the spring by turning the dial by the spring from the zero setting to the 20 setting (or vice-versa), and repeat the experiment with the spring set at this new tension.

After you have completed the Report section, answer the questions on the next two pages.

Part 2. Questions About Circular Motion

Answer the following questions using the quantities obtained in Procedure 1. Answer in complete sentences or equations, such as:

$$W = mg = (\text{value for mass}) \times (9.8 \text{ m/sec}^2) = (\text{value for weight}).$$

1. What is the weight (W) of the revolving body of mass (m)?
2. When the mass (m) is being spun around, what is the centripetal force (F_c) on it (magnitude and direction)?
3. What causes the centripetal force, that is, what physical body or agent exerts the force necessary to keep the object moving in a circle?
4. What causes the spring to be stretched? (HINT: consider Newton's Third Law)
5. From the point of view of an observer riding on the revolving frame, however, this mass behaves exactly as if it were in an effective gravitational field with an acceleration due to gravity (g'). Recall that the revolving mass stretched the spring in our experiment exactly like a heavy weight did, and the mass would appear stationary ($a = 0 \text{ m/s}^2$) to such a rotating observer. Therefore the observer would see the mass pulling "down" on the spring with a weight (W') that is equal in size but opposite in direction to the centripetal force that we observe in the laboratory frame of reference:

$$\vec{W}' = m\vec{g}' = -\vec{F}_c = -m\vec{a}_c$$

From this point of view, what is the effective weight (W') of the revolving mass (magnitude and direction)?

6. What is the value of this g' (magnitude and direction)?
7. Using the same radius as you used above, what frequency (f) expressed in rps and rpm would the mechanism have to rotate at to make the effective weight equal to 100 times the stationary weight, i.e., $W' = (100)W$?
8. Suppose the spring broke while the mass was being rotated. Also assume that the mass flies off and is not immediately stopped by the end of the rotating mechanism, and neglect the effect of the real gravity in the room. What path would the mass appear to take as seen by an observer in the room?
9. What path would the mass in (8) above appear to take as seen by the observer located on the revolving frame? At first, confine your analysis to the time immediately following the break. Then extend your analysis to later times.
10. a) Draw a diagram showing the positions of the end of the rotating bar and the flying mass as seen from the lab frame's point of view at the following five times:
 - t_1 : the instant the mass starts to leave the end of the rotating bar,
 - t_2 : the instant when the bar has rotated 90° from the release point,
 - t_3 : the instant when the bar has rotated 180° from the release point,
 - t_4 : the instant when the bar has rotated 270° from the release point,
 - t_5 : the instant when the bar has rotated a full 360° from the release point.

b) Draw a diagram showing the position of the mass relative to the end of the bar as seen from the point of view of an observer riding on the end of the rotating bar for each of the five times in part (a) above.

11. Finally, with your instructor, run the computer program on centripetal motion and compare your sketches of the motion (question 10) with those generated by the computer, and see if your answers to question 10 are correct. If the computer is off, turn it and the monitor on. Load the Physics Menu. From the menu, type **L** to bring up the **Lab** menu and then enter **3** to run the centripetal force routine.

This idea of effective gravity is the principle that explains the operation of a "centrifuge". The term "centrifugal force" is sometimes given to what we have called the effective gravity. Hence the name centrifuge. Note that centrifugal force is not a real force and does not belong in the $\sum \vec{F}$ term of $\sum \vec{F} = m\vec{a}$. However, to an observer on the rotating frame it appears that centrifugal force is real. Such an observer does not measure the same acceleration as the stationary observer measures and so must *invent* a force, the fictitious "centrifugal force", to account for the difference in acceleration.