1. [30 points] Find integers $r$ and $n$ such that \[ \binom{10}{5} + 2 \binom{10}{6} + \binom{10}{7} = \binom{n}{r} \] (Hint: Pascal’s Identity).

2. [35 points] A combination lock has 22 numbers on its “face”.
   (a) How many different three number permutations can be made if the numbers can be repeated?
   (b) How many different three number permutations can be made if the three numbers are all different (no repetitions)?

3. [35 points] Prove, using proof by induction, that the sum of the first $n$ integers squared is \( \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \). Label the steps in your proof. Hint: Before you begin the induction conclusion step, it may help to substitute $n+1$ for $n$ in \( \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \) so that you can see the “goal” of the induction conclusion step.