Name:

100 points. Points for each question are in [brackets] to the right of the question number or letter. If you use the back of a sheet, please write "on back". CIRCLE ANSWERS. IMPORTANT: show your work and the method you used to work each problem. You may leave answers in combination or factorial notation. You may leave probabilities as fractions. Do not leave "non-executed" multiplications, divisions, subtractions or additions—points will be deducted in such cases. Please ask questions if you do not understand the instructions.

1. [12 points] Find integers \( r \) and \( n \) such that \( \binom{8}{3} + \binom{8}{4} + \binom{9}{5} = \binom{r}{n} \).

\[
\binom{8}{3} + \binom{8}{4} + \binom{9}{5} = \binom{r}{n}
\]

\[
10 = \binom{r}{n}
\]

\[
r = 10, \quad n = 5
\]

2. A batch of 40 components contains 5 defective components and 35 non-defective components. A quality inspector randomly selects a sample of 8 components \textbf{without replacement}.

(a) [5 points] What is the name of distribution of the random variable corresponding to the number of defectives in the sample of 8 components?

\[\text{Hypergeometric distribution}\]

(b) [10 points] What is the probability that the sample includes at least two defective chips?

\[
P(X \geq 2) = 1 - P(X \leq 1)
\]

\[
\frac{n!}{(n-x)!x!} \left[ 1 - \left( \frac{\binom{5}{x}(\frac{40-x}{8})}{\binom{40}{8}} \right) \right]
\]

\[
= 1 - \left[ \frac{(\binom{5}{3})(\binom{35}{1})}{\binom{40}{3}} \right] - \left[ \frac{(\binom{5}{2})(\binom{35}{2})}{\binom{40}{5}} \right]
\]

\[
= 1 - \left[ \frac{0.30603889842}{0.2561288357} \right] - \left[ \frac{0.1437198339}{0.2561288357} \right]
\]

\[
P(X \geq 2) \approx 0.0436203879
\]
3. [12 points] If \( P(A^c \cap B) = 0.15 \) and \( P(A^c \cap B^c) = 0.25 \), find \( P(A) \).

\[
P(A) = 1 - 0.25 = 0.75
\]

\[
P(A) = 0.75 - 0.15 = 0.6
\]

4. An urn contains 60 blue marbles and 15 green marbles. A machine randomly selects four marbles one at a time replacing each marble in the urn after determining the color of the marble.

(a) [5 points] What is the name of distribution of the random variable corresponding to the number of green marbles in the four selected marbles?

Binomial Distribution

(b) [10 points] What is the probability of at least one green marble in the four marbles selected?

绿色概率：\( P_{\text{green}} = \frac{15}{75} = 0.2 \)

\[
P(X \geq 1) = 1 - P(X = 0)
\]

\[
= 1 - \left[ \binom{4}{0} (0.2)^0 (1-0.2)^4 \right]
\]

\[
= 1 - \left[ 1 \times (0.2)^0 (0.8)^4 \right]
\]

\[
P(X \geq 1) = 0.5904
\]

\[
\text{No}
\]
5. A survey asked students 300 students if they read *Time*, *Newsweek* and/or *U.S. News and World Report* (USNWR). Among these students,

- 92 read *Time*,
- 98 read *Newsweek*,
- 100 read USNWR,
- 35 read *Time* and *Newsweek*,
- 33 read *Time* and USNWR,
- 24 read *Newsweek* and USNWR,
- 18 read *Time*, *Newsweek*, and USNWR.

(a) [8 points] Draw a Venn diagram for the frequencies. Include the frequency of students reading none of the three magazines.

\[
300 - 216 = 84
\]

(b) [8 points] What is the probability that a randomly selected student reads exactly two of the magazines?

\[
17 + 15 + 6 = 38
\]

\[
\frac{38}{300} \approx 0.126666667
\]
7. [15 points] Suppose that the number of misprints in a book is a Poisson random variable with an average of 3 misprints per 10 pages. What is the probability of at least 5 misprints in 20 pages?

\[
P(X \geq 5) = 1 - P(X \leq 4)
\]

\[
\lambda = 3\\
w = 2
\]

\[
P(X \geq 5) = 1 - \left[ \sum_{x=0}^{4} \frac{e^{-\lambda} (\lambda^x)}{x!} \right]
\]

\[
P(X \geq 5) = 1 - \left[ \frac{e^{-6} (6)^0}{0!} + \frac{e^{-6} (6)^1}{1!} + \frac{e^{-6} (6)^2}{2!} + \frac{e^{-6} (6)^3}{3!} + \frac{e^{-6} (6)^4}{4!} \right]
\]

\[
P(X \geq 5) = 1 - 0.2850565003
\]

\[
P(X \geq 5) \approx 0.7149434997
\]
Extra credit [3 points]: Using the definition of the expected value and the probability moment function prove that the expected value of a Poisson random variable is $\lambda$.

Prove: $E(X) = \lambda$

$$E = \sum_{k=0}^{\infty} k \frac{(e^{-\lambda})^k}{k!}$$

$0^{th}$ term is $0$

$$E = \sum_{k=1}^{\infty} \frac{\lambda^k}{k!} e^{-\lambda} \lambda^x$$

$$E = \sum_{k=1}^{\infty} \frac{1}{(x-1)!} e^{-\lambda} \lambda^x$$

$$E = \sum_{k=1}^{\infty} \frac{\lambda}{(x-1)!} e^{-\lambda} \lambda^{x-1}$$

Use $\gamma = x - 1$

$$E = \lambda \sum_{k=1}^{\infty} e^{-\lambda} \frac{\lambda^y}{y!}$$

$$E = \lambda (1)$$

$$E = \lambda$$