**Venn Diagrams**

**Dfn** Complement of $A$

$$A^c = \{ x \mid x \in U \text{ and } x \notin A \}$$

$vA$ and $\bar{A}$ are other notations for complement

**Dfn** Union: $A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$

*inclusive or*  

**Dfn** Intersection: $A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$

**Venn Diagrams**

- $A \cup B$
- $A \cap B$
- $A^c$
- $A \cup B$
Disjoint sets A and B

\[ A \cap B = \emptyset \]

Def.

\[
\bigcap_{n=1}^{\infty} A_n = \{ x \mid x \in A_n \text{ for all } n \in \mathbb{N} \}
\]

Thm 2.2: De Morgan's Laws

a) \((A \cup B)^c = A^c \cap B^c\)

b) \((A \cap B)^c = A^c \cup B^c\)

Proof (a):

If \(x \in (A \cup B)^c\), then \(x \notin A \cup B\).

So, \(x \notin A\) and \(x \notin B\). So, \(x \in A^c\) and \(x \in B^c\).

Therefore, \(x \in A^c \cap B^c\).

\[ x \in (A \cup B)^c \iff x \in A^c \text{ and } x \in B^c \]

\[ x \in A^c \iff x \in A^c \cap B^c \]
Thm 2.3: Inclusion-Exclusion Principle

Assume $A, B$ are finite sets.

Then

a) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

b) if $A, B$ are disjoint, then $n(A \cup B) = n(A) + n(B)$

c) if $A \subseteq B$ then $n(A) \leq n(B)$

Dfn: Cartesian product of $A$ and $B$

$A \times B = \{(a, b) | a \in A, b \in B\}$

ordered pair

Thm 2.4

If $A, B$ finite

$n(A \times B) = n(A) \cdot n(B)$
How many 12-digit permutations of 1, 2, 3 are there?

Tree diagram

Since order is relevant for permutating, there are 9 permutations.
Thm 3.1 Fundamental Theorem of Counting

If a choice consists of $k$ steps of which the first can be made in $n_1$ ways, the second in $n_2$ ways, ..., and the $k$th can be made in $n_k$ ways, then the choice can be made in $n_1 \cdot n_2 \cdot \ldots \cdot n_k$ ways.

Proof p. 32
<table>
<thead>
<tr>
<th>Dfn</th>
<th>How many ordered arrangements of 3 elements: $a, b, c, d$?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>permutation: an ordered arrangement of $n$ elements, or an ordered $n$-tuple.</td>
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\[ P(n, k) = \frac{n!}{(n-k)!} \]

Why "distinct" elements
EX  How many permutations of letters in "home"?

4!

How many permutations when "m" is second letter?

If "m" is 2nd letter, then

3 choices for letter 1
2 choices for letter 3
1 choice

So, 3!

How many ways if letters in 2 positions are fixed? 2! = 2

How many ways if letters in 3 positions are fixed
The combination is a subset — order does not matter — of distinct elements.

\[ C(n,k) = \frac{P(n,k)}{n!} = \frac{n!}{k!(n-k)!} \]

**Example:** From a group of 5 Engineering majors and 8 Computer Sci majors, how many committees of 2 Eng majors and 2 Computer Sci majors can be formed?

\[ \binom{5}{2} \cdot \binom{8}{2} = \frac{5!}{2!3!} \cdot \frac{8!}{2!6!} = \frac{5 \cdot 4 \cdot 3}{2 \cdot 1} \cdot \frac{8 \cdot 7 \cdot 6}{2 \cdot 1} = 280 \]